Fractal Characteristics of Rayleigh-Taylor Unstable Systems

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Introduction

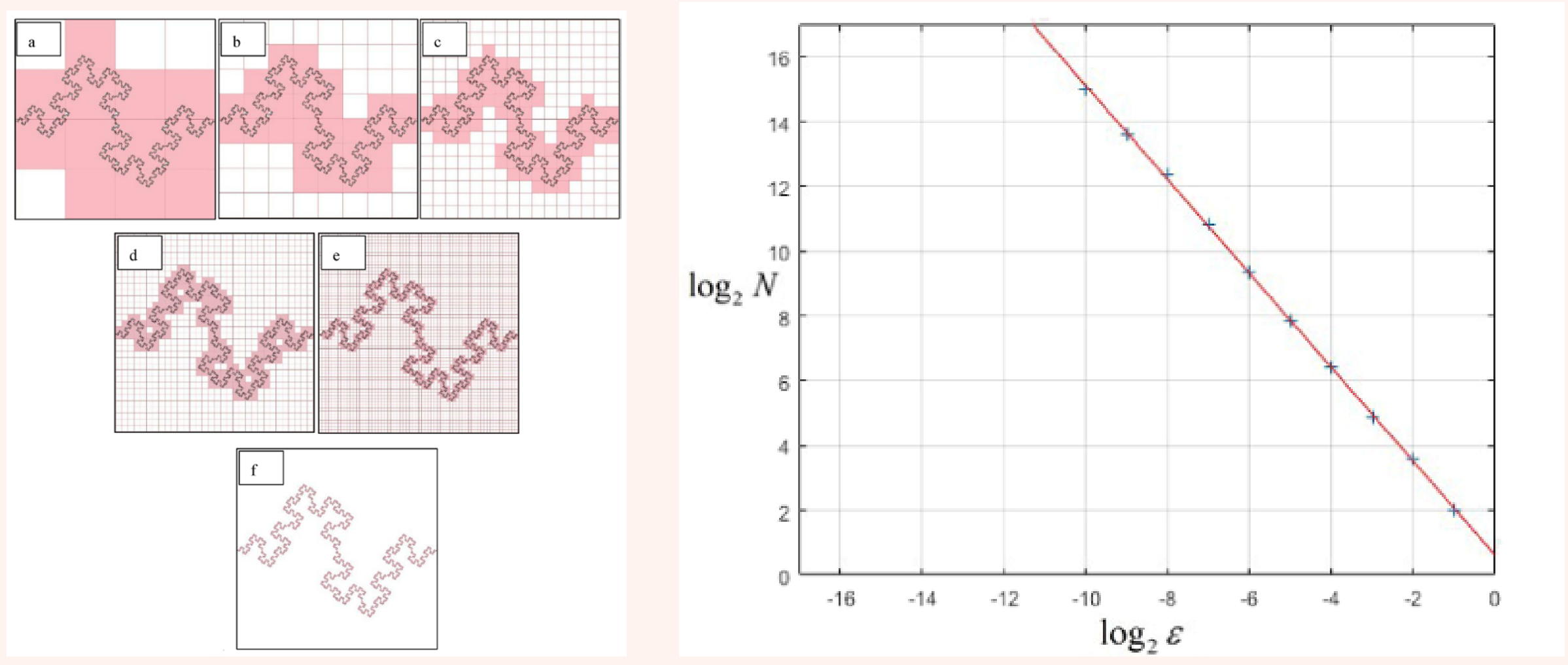
Thermonuclear supernovae, particularly Type Ia supernovae (SNe Ia), are among the most energetic and cataclysmic events in the universe. These stellar explosions result from the runaway thermonuclear burning process in the dense degenerate material of a white dwarf star accreting matter from a non-degenerate companion. Not only do they generate interest as bright and visible objects, but SNe Ia also serve as standardizable cosmological candles, enabling precise measurements of cosmic distances and probing the expansion history of the universe.

The Rayleigh-Taylor Instability (RTI) is believed to play a significant role in the early explosive phase of certain physical processes, such as the ignition and propagation of flames. Multi-dimensional simulations have shown that the burning rate associated with the laminar flame is insufficient to explain observations, motivating the consideration of turbulence driven by RTI as a necessary ingredient. This turbulence is associated with the flame evolving in the gravitational field of the white dwarf or companion star. When an explosion occurs, it results in an unstable flame front that grows in size. This flame front represents the interface between unburnt fuel and burnt ash, across which a density change occurs. The RTI theory suggests this density interface is subject to instability, leading to folding and growth of the flame front. This growth in surface area can result in flame acceleration, a crucial factor for models aiming to better represent the observed characteristics of these explosive phenomena. Considering the role of RTI holds promise for producing models that more closely reflect the fundamental physics with characteristics similar to observations.

In this study, we consider the hypothesis proposed by Blinnikov et al. (1995) that the fractal characteristics of RTI-induced flame surfaces substantially affect flame acceleration in SNe Ia. We investigate the fractal properties of physics systems, incorporating varying physics, to examine the persistence of scaling behaviors and to assess how differences in physics influence Rayleigh-Taylor (RT) unstable models.

Computational Method

We are estimating the fractal properties of our models using the method of box counting in the models considered here. This method relies on the assumption that complexity scales with the richness of structure, suggesting a certain type of scaling at different scales.



(a) (b)

Figure 1. (a) (left panel) the number N of boxes changes with different side lengths ϵ. (b) (right panel) the linearly fit slope with the least square method. Ten different points of different N and ϵ are selected in (a), and ϵ is taken as a power law.

We first obtain data on a grid, using box counting to quantify the interface-cut cells within the mesh. This process includes decimating the mesh (ϵ), recounting affected cells ($N(\epsilon)$), and thus establishing a scalable relationship.

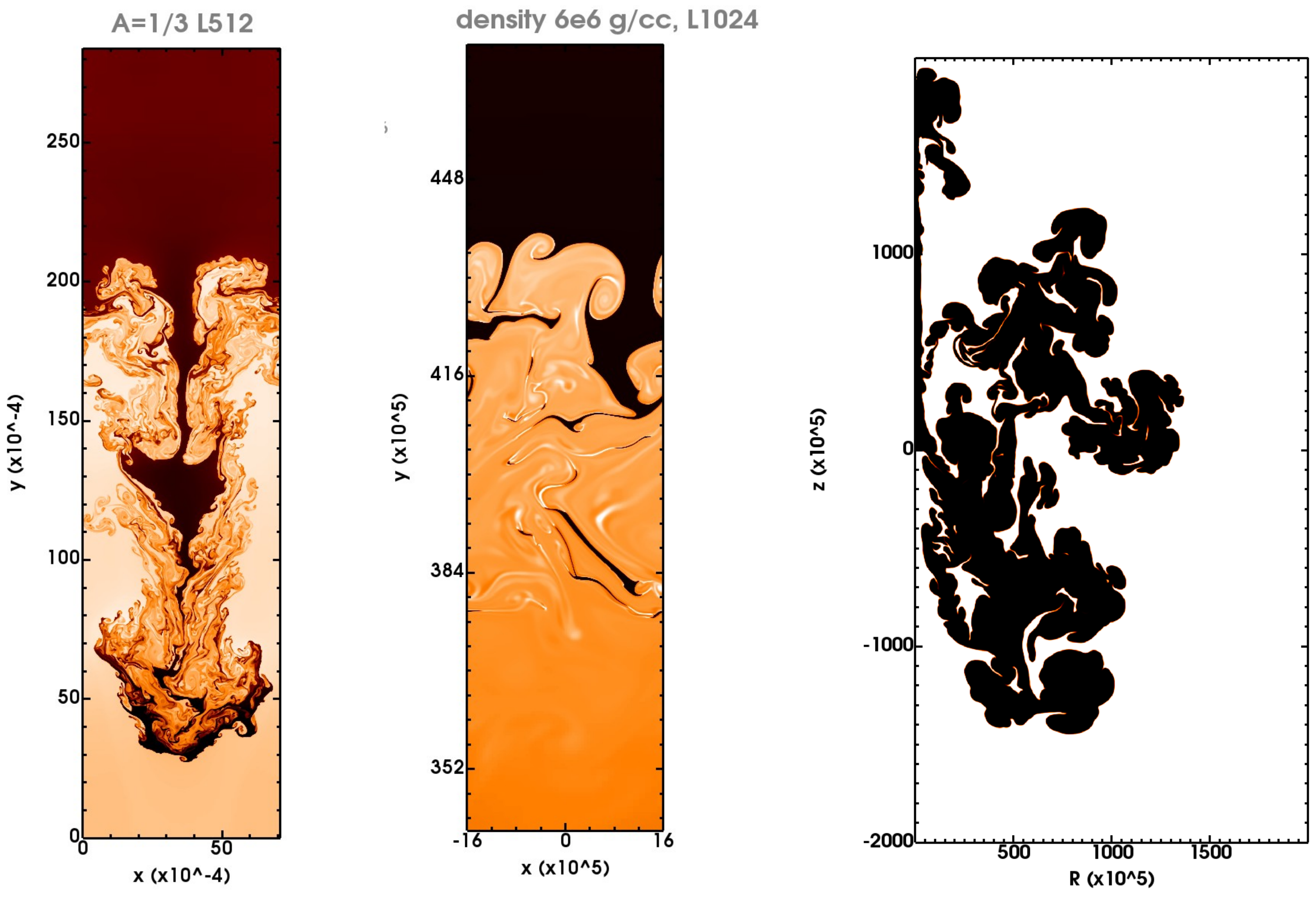
The fractal dimension (D) is estimated through the relationship

$$D = \lim_{\epsilon \to 0} \frac{\log(N(\epsilon))}{\log(1/\epsilon)}$$

Simulation and Data

In the context of flame growth and SNe Ia, we investigate the fractal properties in the following models:

1. Pure Hydrodynamic RTI Model (RTH): In this model, we consider material discontinuity between two different fluids subjected to constant gravity. As such, this model represents the simplest configuration that displays the RTI behavior.
2. Rayleigh Taylor Flame Model (RTF): This model is similar to the RTH model, but incorporates flame physics and employs Khokhlov's flame-capturing algorithm to describe the flame propagation under conditions typical of low-density layers in white dwarfs. We analyze the propagation of RTI unstable flames under specific density, gravity, and composition found in explosion models, particularly in supernova SNe Ia scenarios.
3. Supernova Explosion Model (RTSN): This integrated model includes the same physics as the RTF model but accounts for realistic conditions found in the stellar evolution models of massive white dwarfs, including self-gravity effects and variable composition.



(a) (b) (c)

Figure 2. (left panel) RTH: density distribution at the final simulation time for the model with an interface separating heavy and light material, with a density ratio of 2 (Atwood number = 1/3). (middle panel) RTF: density distribution at the final time when the model reached some quasi-steady state. (right panel) RTSN: flame progress variable. A key distinction lies in the interface structures. In the RTH model, the density distribution exhibits pronounced mixing, resulting in a poorly defined material interface. Conversely, in the other two models, although the surface of the flame appears heavily corrugated, the flame front demonstrates a clearly defined structure [see text for details]

Simulation and Data cont.

Simulation data for these models were obtained using the Proteus code as a custom version of the University of Chicago Flash Code (Fryxell et al. 2020), which is massively parallel, multiphysics, adaptive best requirement, and finite volume code. We analyze these models in terms of their fractal characteristics, expecting to uncover potential universal scaling associated with RTI. Despite employing diverse physics in the models under investigation, our primary focus is on establishing a connection to the core motivation, wherein RTI-induced turbulence emerges as a crucial factor. By examining the fractal characteristics, we aim to understand how these features impact flame acceleration in SNe Ia

The data consists of a large number of snapshots taken throughout the simulation time, providing high resolution per wavelength for all models. These snapshots enable us to study the evolution of the fractal dimensional model effectively. Therefore, it is more pertinent to characterize the models based on relevant analysis methods and the abundance of time slices available for studying their evolution.

Results and Discussion

Showcasing the evolution of fractal dimension over time in RTH at different resolutions, Figure 3 illustrates the temporal evolution of interface complexity. Three curves correspond to different simulation resolutions: RTH-128, RTH-256, and RTH-512. The models all demonstrate an increase in fractal dimension from initially smooth conditions, progressing rapidly into highly nonlinear evolution. While there may be evidence of saturation in the fractal dimension, generally, they closely follow each other without systematic differences in later stages. The initial differences in fractal dimension are attributed to the resolution of the initial conditions, as box-counting with varying initial scales leads to different interface appearances. However, as the systems enter a strongly nonlinear regime, differences diminish, suggesting a convergence towards a relaxed state.

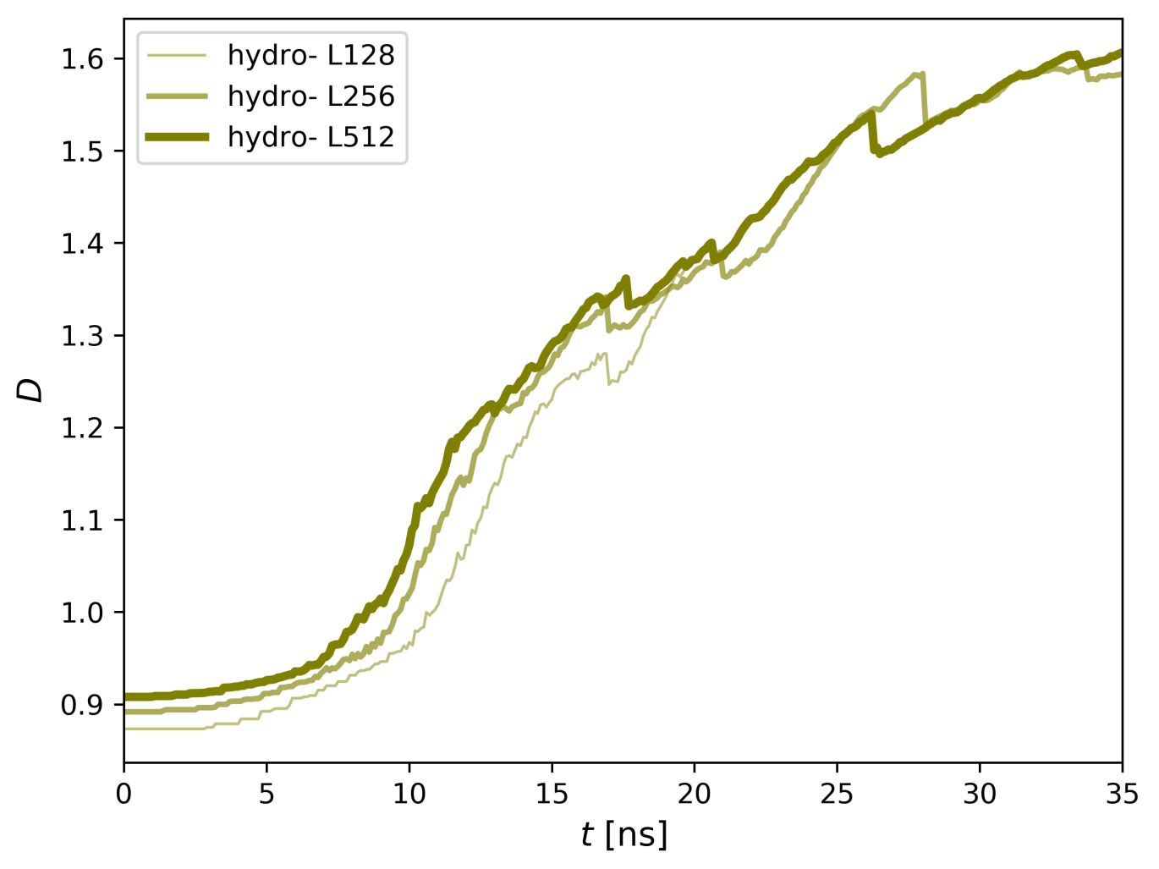


Figure 3. temporal evolution of fractal dimension in RTH

The fractal dimension evolves from a value close to 1 in Figure 4 in RTF models, similar to the behavior observed in RTH models, reflecting the slightly perturbed initial interface conditions. It rapidly increases and eventually levels off at a value of approximately 1.2. As the dynamics enter a nonlinear regime, the fractal dimension curves exhibit a stabilizing behavior, suggesting a saturation in the degree of flame surface complexity. The observed highly oscillatory behavior may signify rapid small-scale changes within the model.

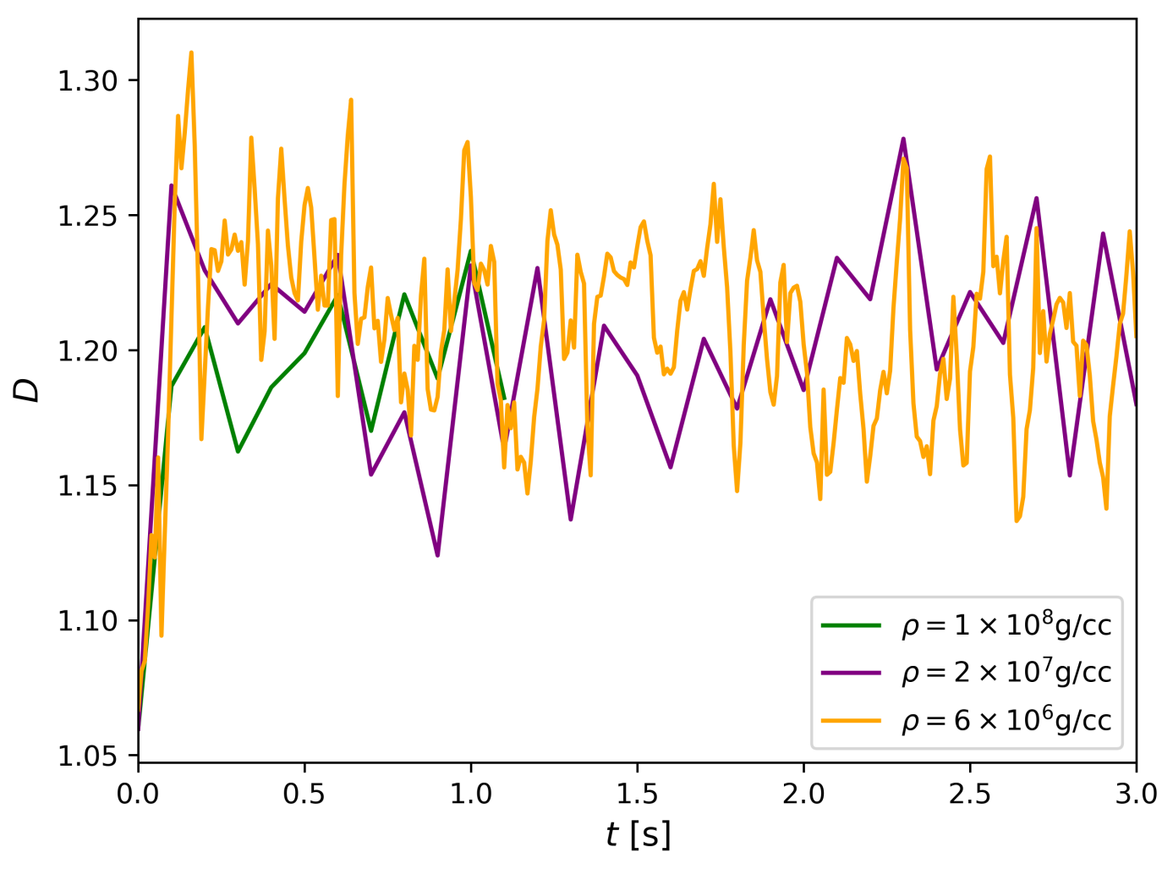


Figure 4. temporal evolution of fractal dimension in RTF

The results depicted in Figure 5 showcase variations in fractal growth among the three RTSN model realizations, each with slightly differing initial conditions. There is an abrupt change in fractal dimension around 0.8, potentially due to the early stages of flame evolution. The initial growth of the flame region, analytically done on the mesh, leads to a temporary absence of real flame evolution. This may be attributed to changes in mesh resolution as the flame reaches a certain radius, leading to fluctuations in fractal dimension. However, despite these fluctuations, all realizations converge to a similar fractal dimension, indicating their resemblance to RTF.

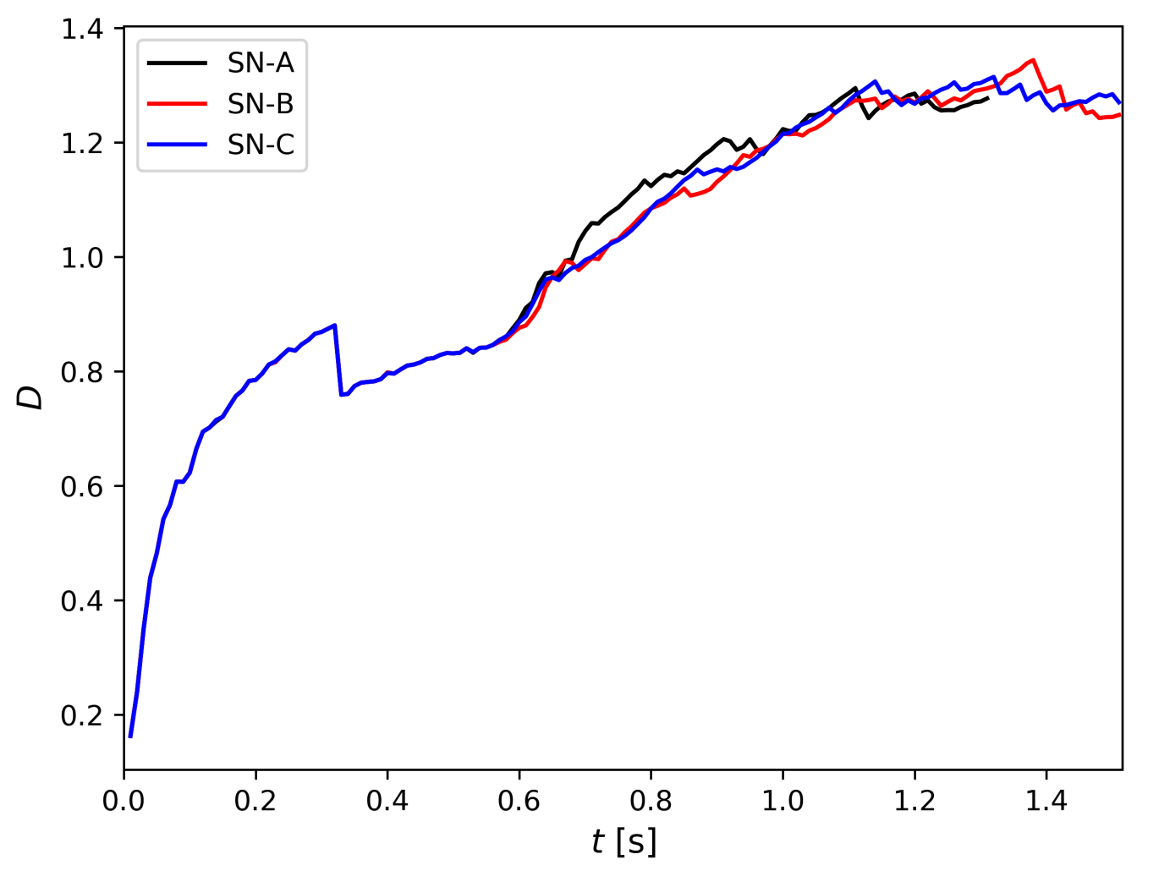


Figure 5. temporal evolution of fractal dimension in RTSN

The analysis of our simulations across the three different models reveals both similarities and differences in the evolving fractal dimension characterizing the interface structures. Initially, the fractal dimension increases from low initial values as the interface becomes increasingly convoluted across all systems, indicating a commonality in the development of complex structures driven by RTI. However, the maximum attainable fractal dimension differs between RTH, RTF, and RTSN. In the RTH case, the continuous destruction of the interface contrasts with the restoration process in our RTF and RTSN models, influencing their appearance and complexity. Coherent flame structure breaks down in the RTH model, leading to a disrupted, smeared interface once the instability grows sufficiently strong.

Recommendations

The future extension of this work is to consider three-dimensional situations because the actual physical systems are three-dimensional, and we know that processes responsible for the evolution of the rate of stability, particularly RTI-driven turbulence, are inherently three-dimensional. Emphasizing the significance of the transition offers a promising way to explore the complex dynamics of RT unstable systems. This approach aligns with the foundational insights provided by Blinnikov et al. (1995) regarding the scaling laws and fractal characteristics. We seek to connect the fractal characteristics observed in well-resolved flame models to the under-resolved supernova explosion models.

References

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