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Multiphysics laser-driven high-energy density experiment design: Initial multidimensional results

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Abstract : A challenge facing simulations of the coupling of hydro and thermal conduction in a high-energy density regime is an accurate approximation of the thermal conduction coefficient. We will use the recent theory of conduction-mediated Rayleigh-Taylor instability proposed by our group and combine it with 1-D and 2-D hydrodynamic simulations to assess the feasibility of the proposed experiments. More specifically, the experiment will be based on an initial configuration where thermal conduction can make a system Rayleigh-Taylor unstable whereas in the absence of thermal conductivity, the system would be stable. This scenario can exist in an accelerating medium with a graded chemical composition (e.g., in an alloy with spatially varying composition, or mixture of a spatially varying concentration of the components) and an applied temperature gradient. By introducing seed perturbations in the initial conditions, and observing their growth as the unstable system is set into accelerated motion, one can directly infer a coefficient of thermal conductivity. A subset of thermodynamic properties can also be obtained, if measurements are performed at several wavelengths. The actual experiment would consist of three parts: (1) manufacturing an experimental package, (2) driving the package in such a way as to create the required temperature profile, and (3) subjecting the package to a protracted acceleration during which the theoretically predicted instability would develop and be observed. Here, we are testing this theory developed by Dmitri Ryutov for measuring thermal conduction coefficients in this type of regime. Our simulations test Ryutov's perturbation analysis through simulation which will be important for actual experiment design in fields such as laboratory astrophysics. Here, we probe initial conditions to find the appropriate ranges for design parameters to capture Rayleigh-Taylor like instabilities driven by thermal conduction.

Measuring Thermal Conductivity

Idea: A big challenge in accounting for thermal conductivity in RT simulations in an HED regime is having an accurate measurement of the thermal conduction coefficient κ. We are testing a theoretical technique for making accurate measurements of the thermal diffusivity which provides us directly with the thermal conduction coefficient κ.

In [2], Ryutov points towards a technique for taking measurements of the thermal conduction coefficient experimentally. Ryutov points out that in the laboratory one could create a situation where the classical RT perturbations are stable whereas those driven by thermal conductivity are unstable. By embedding a known perturbation in the ICF design target one can observe it's growth in a standard way, through a laser driven experiment. As he has shown, that instabilities exist only with finite thermal conductivity, then the growth rate will be a direct measure of the thermal conductivity coefficient through the use of the dispersion relation:

$$\frac{1}{\Gamma^2}\frac{gk^2}{k^2+q^2}\left(\frac{\rho'}{\rho}+\frac{g}{s_{AD}^2}\right)-1=\frac{\chi^2(k^2+q^2)}{s_{AD}^2\Gamma}\left(1+\frac{p_C}{s_{IS}^2\rho}\frac{gC'}{\Gamma^2}\frac{k^2}{k^2+q^2}\right) \tag{1}$$

where s_{AD} and s_{IS} are an adiabatic and isothermal sound speeds, g is the acceleration, C is the compositional constant, and Γ is the complex RT growth rate. Instead of thermal conductivity κ we have introduced the thermal diffusivity χ,

$$\chi=\frac{\kappa}{\varepsilon_T}. \tag{2}$$

The dispersion relation given by Eq. (1) involves quantities either known analytically or that can be calculated throughout an experiment or simulation leaving the only unknown to be the thermal diffusivity, thus giving us a direct method to calculate the thermal conductivity of the material medium with which we are working.

If we consider (1) in the limit as the thermal diffusivity χ goes to zero, one is able to recover the dispersion relation for adiabatic perturbations,

$$\Gamma^2=\Gamma_{AD}^2\equiv\frac{gk^2}{k^2+q^2}\left(\frac{\rho'}{\rho}+\frac{g}{s_{AD}^2}\right). \tag{3}$$

On the other hand, if we consider the limiting case of very high thermal diffusivity we recover a different class of perturbations, in the isothermal regime, who's stability is determined by the sign of the product $p_C C'$:

$$\Gamma^2=\Gamma_{IS}^2\equiv-\frac{gk^2}{k^2+q^2}\frac{p_C C'}{\rho s_{IS}^2}. \tag{4}$$

The quantities Γ_{AD} and Γ_{IS} represent growth rates in the adiabatic and isothermal regimes respectively. In general, the quantity Γ which is determined via (1) is not strictly within either an adiabatic or an isothermal regime. This dispersion relation shows us that in the adiabatic regime we recover the growth rate of a pure hydro or classic Rayleigh-Taylor instability. When the thermal diffusivity is not negligible then the quantity Γ ceases to behave like the classic RTI. This is because the effect of thermal conduction on the fluid changes the pure hydro instabilities significantly so that what we observe is the propagation of an entirely new type of instability.

A Model Problem

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x_j}(\rho v_j)=0 \tag{5}$$

$$\frac{\partial}{\partial t}(\rho v_i)+\frac{\partial}{\partial x_j}(\rho v_i v_j)=-\frac{\partial p}{\partial x_i}+\rho F_i \quad (i=1,2,3) \tag{6}$$

$$\frac{\partial(\rho E)}{\partial t}+\frac{\partial}{\partial x_j}((\rho E+p)v_j)=\rho v_j F_j, \tag{7}$$

where ρ is the density, p is the pressure, E is the total energy, v are the components of velocity, and F is some external force such as gravity, for example. The first equation represents the conservation of mass, the second conservation of momentum, and the third conservation of energy.

We couple the fluid equations with thermal conductivity by solving a heat equation and adjusting the energy equation as follows,

$$\frac{\partial(\rho E)}{\partial t}+\frac{\partial}{\partial x_j}((\rho E+p)v_j)+\nabla\cdot(\kappa\nabla T)=0. \tag{8}$$

The domain for our problem is a tube with two chemical species, hydrogen and carbon. A perturbation is imposed along the interface between the two species of the form, $A\sin(\omega\bar{x}+\phi)$.

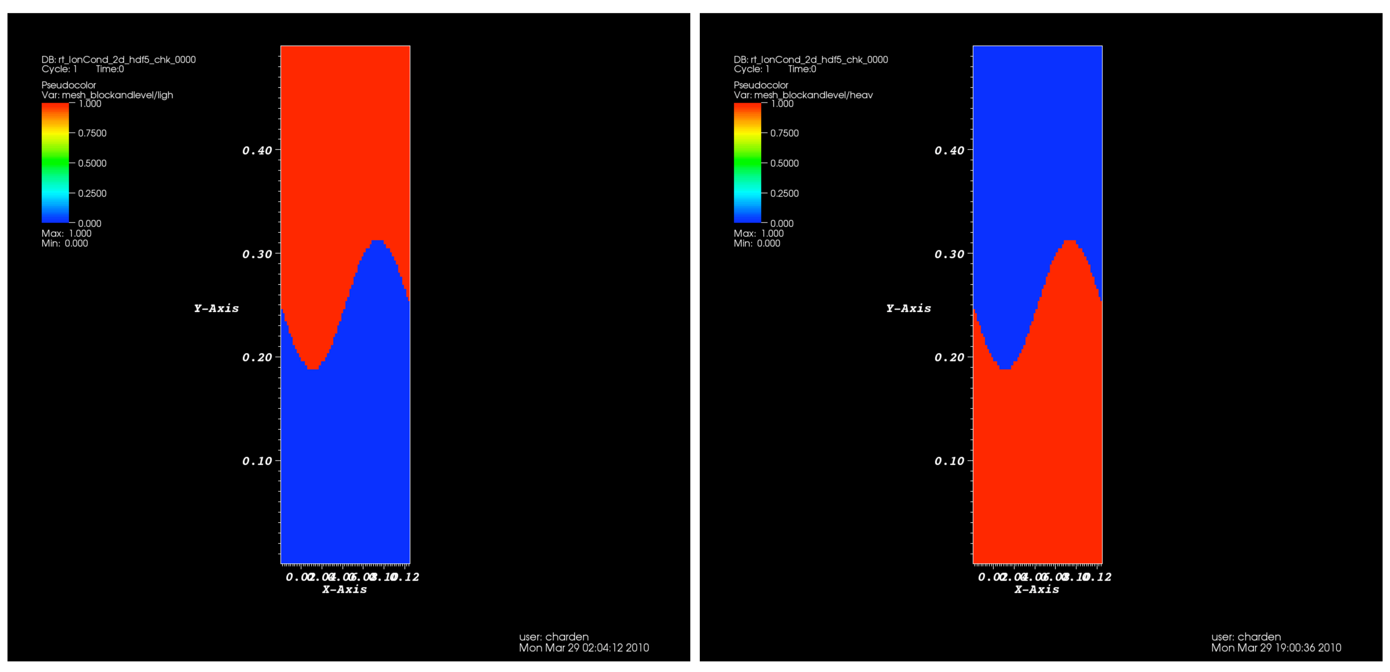

Figure 1: Initial setup of chemical species, with pure hydrogen above the interface and pure carbon below the interface.

This configuration is RT stable in a pure hydrodynamic sense. This means that the the system is already in a steady state and no dynamic evolution of the state variables will occur.

2D Results

Initial Conditions

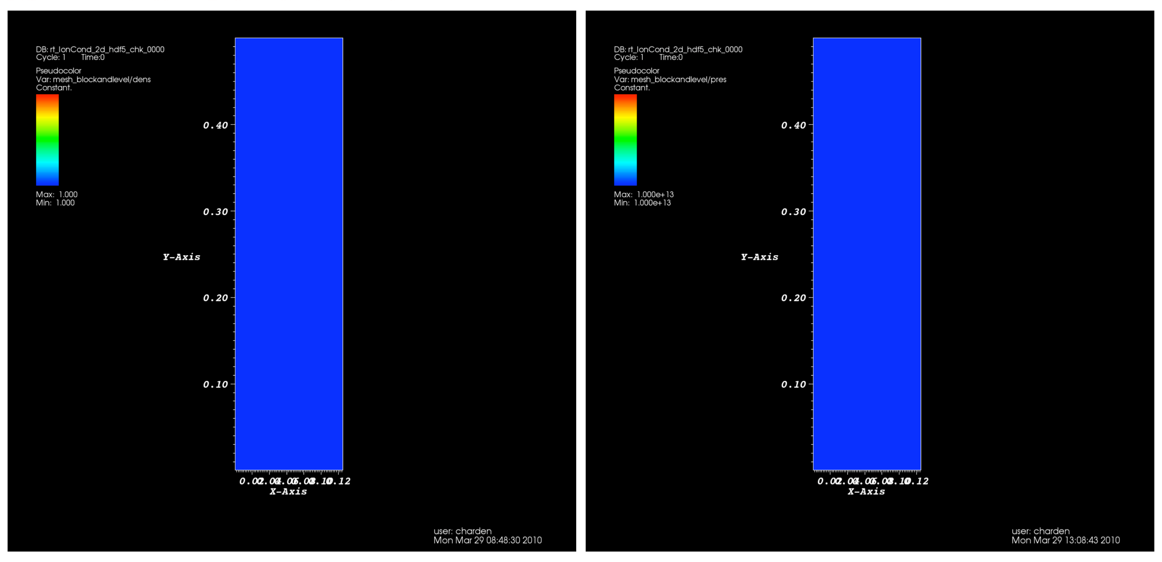

Figure 2: Initial setup of state variables, with $\rho=1.0$ and $p=1e^{13}$.

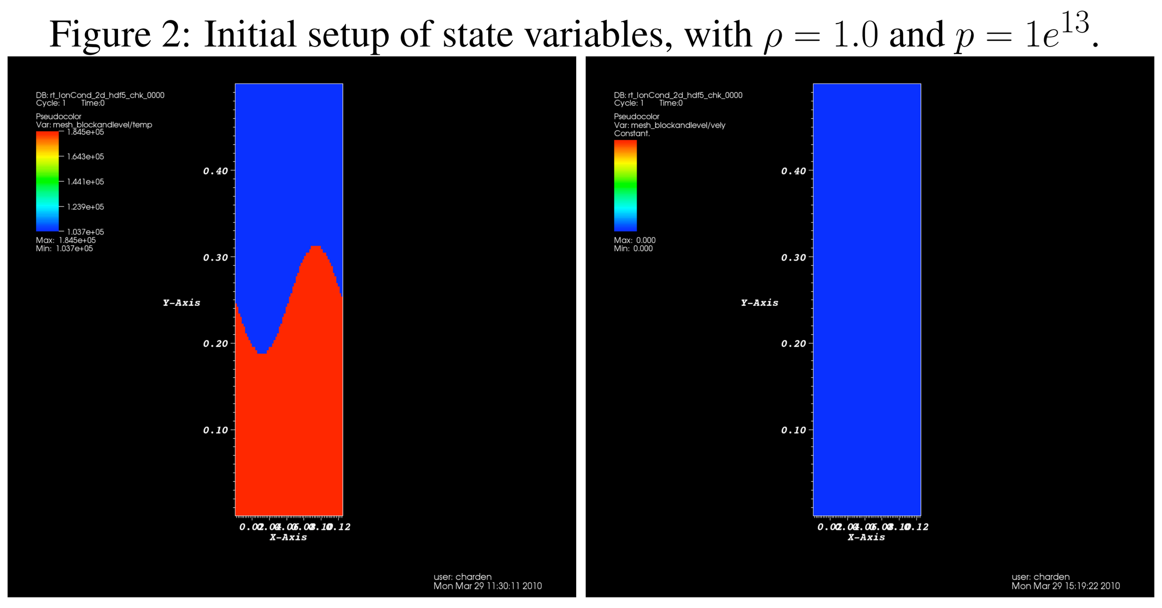

Figure 3: Initial setup of state variables, with $T=1e^5$ and $u_y=0.0$.

Dynamics Due to Thermal Transport

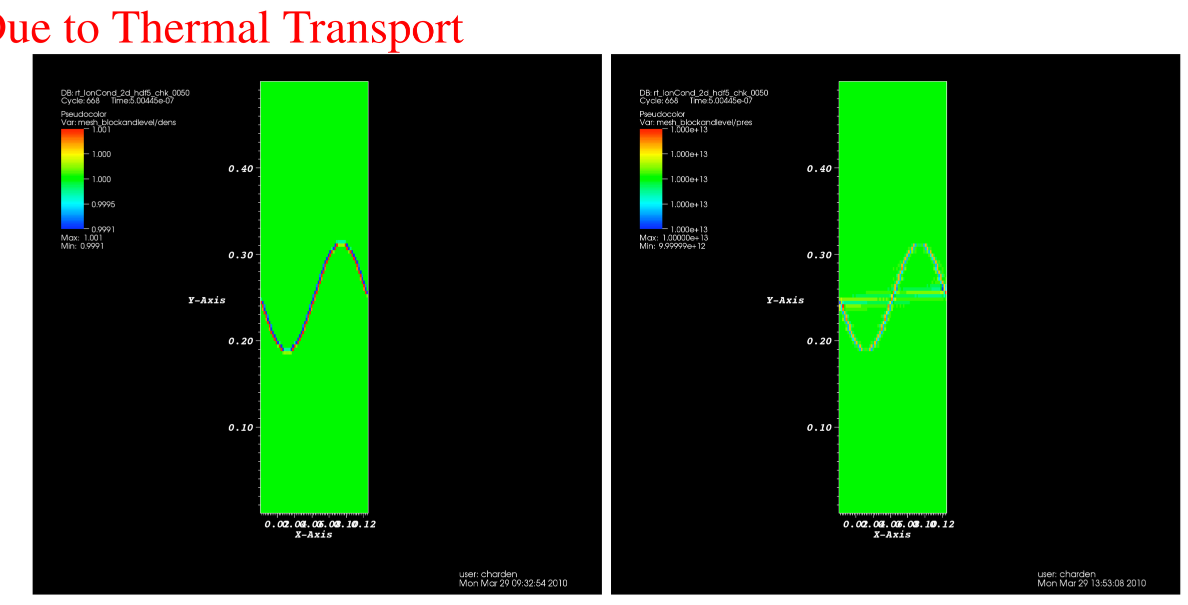

Figure 4: Evolution of state variables ρ and p due to thermal transport.

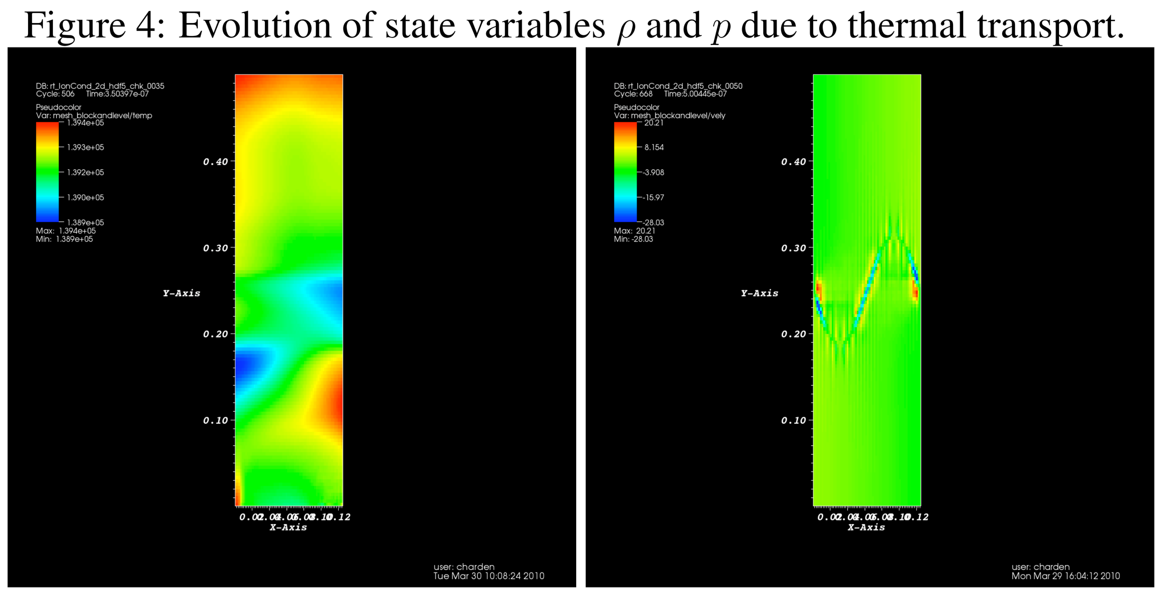

Figure 5: Evolution of state variables T and u_y due to thermal transport.

Here we see that the addition of thermal transport in the hydrodynamic system drives dynamic behavior inducing instabilities in the state variables around the region of the perturbed interface.

Future Work and Sensitivity

The next step in this research is to continue probing initial conditions in 2D and developing domains with graded chemical compositions such as what is shown here in Fig. 6.

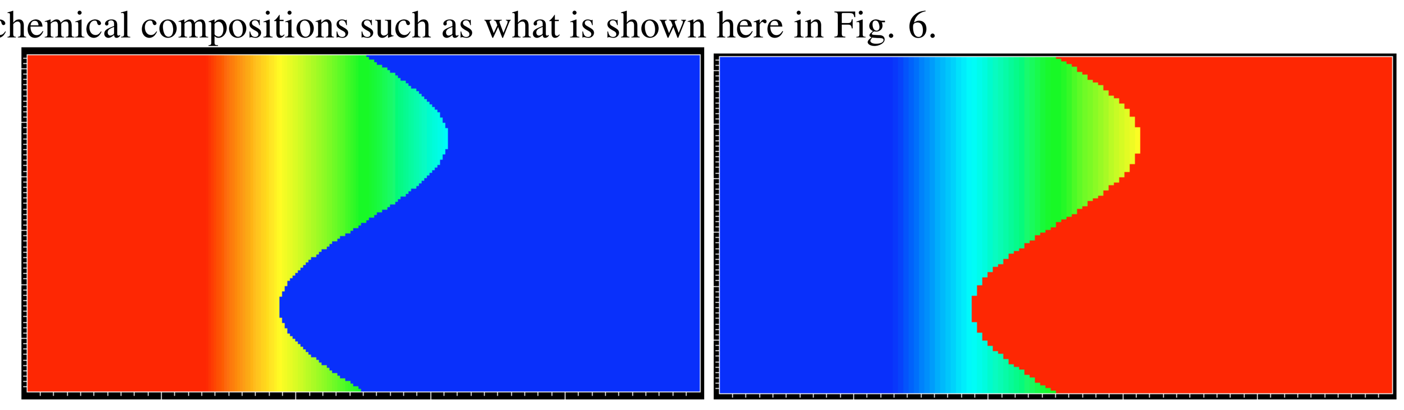

Figure 6: Massfraction of hydrogen is graded linearly as the perturbed interface is approached.

We need to understand and quantify the sensitivity of the dynamics and instabilities that are being driven by thermal transport in terms of the various input parameters that we are able to control within our experiments. The control parameters include terms from Eq.(1) such as the compositional gradient C' and the wave numbers k and q. We may also investigate the sensitivity of the parameters which make up the perturbation imposed at the interface between the chemical species such as the amplitude A, wave number ω, and the phase shift ϕ. We intend to employee a stratified sampling technique such as Latin Hypercube Sampling to perform our sensitivity analysis.

The next stage will involve extending these simulations to 3D. Performing verification, sensitivity analysis, and hopefully getting to the point of laboratory experiment design and validation exercises.

References

[1] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* , Dover Publications, New York, 1981

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[3] David L. Youngs, *Numerical Simulation of Turbulent Mixing by Rayleigh-Taylor Instability.* ,Physica 12D, pp 32-44, 1984