The Babylonian Theory of the Planets

Hugh Thurston

The Babylonians developed sophisticated theories for the motions of the planets. These are interesting not so much for the light that they throw on the planets as for the methods used by modern research workers in interpreting them. Often the results of these researches are given without mention of how they were derived, and sometimes data are described as attested when in fact they do not appear in the Babylonian tablets but were derived from them.

Two types of tablet are concerned with the theories. Some tablets list calculated data. Others, called procedure tablets, explain the calculations.

There are two main theories: a zonal system in which the ecliptic is divided into zones in each of which a relevant quantity is constant, and a zigzag theory, in which a quantity increases at- a constant rate from a minimum to a maximum, decreases at the same rate to the minimum, increases at the same rate, and so on. These (unimaginatively in my opinion) are usually called system A and system B.

Most of the tablets concerned with the planets give calculated dates and longitudes for the synodic phenomena; very few give data for the planet between the phenomena.

The synodic phenomena for an inner planet are its first appearance in the morning, MF; its subsequent disappearance, ML; its appearance and disappearance in the evening, EF and EL;
and its stationary points. (The abbreviations morning first, morning last, and evening first and last, are from van der Waerden.)

The synodic phenomena for an outer planet are its appearance (in the morning) MF; its disappearance (in the evening) EL; opposition; and the beginning and end of retrogression, BR and ER.

The Babylonians recorded the longitude of the planet at each occurrence in signs (of the zodiac) and thirtieths of a sign, which I shall convert into degrees.

The arc of the ecliptic separating two successive occurrences of a synodic phenomenon is a synodic arc.

Many zonal systems are described in procedure tablets. These include a system for Mercury's MF using three zones, a different system for its EF also using three zones; three different systems, each using two zones, for Jupiter, one using four zones and one using six zones; and finally a system for Saturn using two zones.

There are also tablets for the ML and EL of Mercury clearly using zonal systems. We have not found a procedure tablet explaining either of these but the zones can be deduced from the longitudes. The results are often cited, and the deduction must have been made early on, probably by Kugler in a work that I cannot find. (Nor can anyone whom I have asked.) The same applies to a system for Mars using six zones.

The first aim of this paper is to show how the deductions could be made. The second aim is to comment on the relations between the synodic and the sidereal periods.

Zonal system

To show how a typical system works I use the one for the MF of Mercury explained in ACT 801.

Given the longitude X of one MF to find the next:-

First step. If A is between 121° and 286° add 106°

If between 286 ° and 60° add 141° 20'

If between 60° and 121° add 94° 13' 20".

Second step If the arc added takes you into the next zone, multiply the portion in this second zone by the arc added in the second zone divided by the arc added in the first zone.

Example If A. is 201°, the first step takes us to 307°, of which 21° is in the second zone. We multiply this by 4/3 (as stated in ACT 801: of course, 141° 20' is 4/3 of 106°), getting 28°. So the next longitude is 28° in the second zone, which is 314°. (If this had taken us into a third zone, a third step would have been needed.)

Mercury

ACT 300a gives the longitudes of successive occurrences of ML. By subtracting one longitude from the next we find the length of the synodic arc. These arcs, arranged in the order of the longitudes at which Ghey start, are:

start	arc
12°	120°46'40"
31-40'	117°30[1]
43°	US^^O"
5r20'	U4"13'20"
62°40'	ll2°20'
71°	lll^O'
82°20'	m-20'
100°	*m°2o'*
116°23'20"	116"36'40"

start	arc
132"46'40"	*119°53'10"*
149°10'	123-10'
165"33'20"	*lly.<'26'W*
175°	121°36[1]
182"20[1]	120".25'
WW	U7°35'
213-20'	112°40'
233°-	107°45'
252<'40I	102°50'

start	arc
272°20[1]	99°40'
288°	103-40'
296°30'	lOB^O'
302°45'	108"35[1]
311°15[1]	111"25'
317[a]30[1]	113"30[1]
326"	116"20'
340°45'	n9"15'
355°30[1]	120"53'2^"

From 12° to 62°40' there is a steady decrease of 1' in the length of the synodic arc for every 6' increase in the longitude at which it starts. This suggests that the data follow a system like the one just described with an added arc of length X° in a zone covering these longitudes and an added arc of length 5X°/6 in a zone covering the end-points of these synodic arcs, which range from 132°46'40" to 175°.

Similarly, an increase of 1 in 5 from 100° to 149° suggests an added arc of length Y° here and one of length 6Y°/5 from at least 213°20' to 272°20'.

A decrease of 1 in 4 from 182°20' to 252°20' suggests an added arc Z° here and 3Z°/4 from at least 302°45' to 355°30'.

An increase of 1 in 3 from 288° to 326° suggests W° here and 4W°/3 from at least 31°40' to 82°20'.

There is an arc of X° from at least 12° to 82°40' and an arc of 4W°/3 from at least 31°40' to 82°20': Therefore $X = 4W/3$. Similarly $Y = 5X/6$ and $Z = 6X/5$. Therefore $X = Z = 4W/3$ and $Y = 10W/9$.

So we have four zones.
4W°/3 from x° to y°, covering at least 12° to 82°20'
10W°/9 from y° to z°, covering at least 100° to 175°
4W°/3 from z° to w°, covering at least 182°20' to 272°20'
W° from w° to x°, covering at least 288° to 355°30'.

The synodic arc from 12° to 132°46'40" will pass at y° from the 4W°/3 zone to the 10W°/9 zone, so $132°46'40'' = y° + (12° + 4W°/3 - y°)\times 5/6$. Then

$$y/6 + 10W/9 = 122\tfrac{7}{9} \qquad (1)$$

The synodic arc from 71° to 182°20' does not share the steady decrease, so it must pass both y° and z° into the second 4W°/3 zone.

$$y + (71 + 4W/3 - y)\times 5/6 = y/6 + 355/6 + 10W/9.$$

Then

$$182\tfrac{1}{3} = z + (y/6 + 355/6 + 10W/9 - z)\times 6/5$$

so

$$y/5 - z/5 + 4W/3 = 182\tfrac{1}{3} - 71 = 111\tfrac{1}{3} \qquad (2)$$

For the synodic arc starting at 100°,

$$213\tfrac{1}{3} = z + (100 + 10W/9 - z)\times 6/5,$$

so

$$4W/3 - z/5 = 213\tfrac{1}{3} - 120 = 93\tfrac{1}{3} \qquad (3)$$

From (2) and (3) $y/5 = 18$, so $y = 90$. Then, from (1), $10W/9 = 107\tfrac{7}{9}$, giving $W = 97$ and $4W/3 = 129\tfrac{1}{3}$.

From (3), $z/5 = 36$, so $z = 180$.

For the synodic arc from 182°20' to 302°45',

$$302\tfrac{3}{4} = w + (182\tfrac{1}{3} + 129\tfrac{1}{3} - w)\times 3/4 = w/4 + 233\tfrac{3}{4}.$$

Then $w/4 = 69$, so $w = 276$.

For the arc from 296°30' to 43°,

$$43 = x + (296\tfrac{1}{2} + 97 - 360 - x)\times 4/3 = 44\tfrac{2}{3} - x/3$$

So $x/3 = 1\frac{2}{3}$, giving $x = 5$.

To sum up, ML has four zones:

From	5°	to	90°	add	129°20'
From	90°	to	180°	add	107°46'40"
From	180°	to	276°	add	129°20'
From	276°	to	5°	add	97°.

It is unfortunate that Neugebauer described these as zones of constant synodic arc: the synodic arc of Mercury is nowhere constant.

Mars

ACT 501, 502 and 504 give calculated longitudes for BR. If we calculate the synodic arcs and arrange them in the order of the longitudes at which they start we have the following results (starting longitude followed by arc).

15°	50°	115°	30°	210°	60°
18° 45	48° 45	116° 40	30°	228° 40	69° 20
40°	45°	133° 20	34° 26 40	230°	70°
64°	38° 40	135°	35°	233° 20	71° 40
65°	38° 20	145°	38° 20	256° 40	80° 50
67° 30	37° 30	146° 40	38° 53 20	270°	82° 30
85°	31° 40	170°	40°	298°	75° 30
102° 40	30°	183° 20	46° 40	305°	73° 45
103° 40	30°	185° 33 20	47° 46 40	337° 30	62° 30
105°	30°	207° 46 40	58° 53 20	353° 30	57° 30

These fit together so well that it is quite clear that the three tablets use the same system.

From 102° 40 to 116° 40 the synodic arc is 30°. Therefore there is a zone covering the ecliptic at least from 102° 40 to 146° 40 for which the added arc is 30°. From 64° to 85° the synodic arc is reduced by 1° for every increase of 3° in the longitude at which it starts. So here we have a zone with an added arc of which 30° is two-thirds. The added arc here is therefore 45°. To fall from 31° 40 at 85° to 30° needs a reduction of 1° 40 in the arc and therefore an increase of 5° in the longitude. Therefore the 30° zone begins at 90°.

From 133° 20 to 146° 40 the arc increases by 1° for every increase of 3° in longitude, so in this zone the added arc is 4/3 of 30°, i.e. 40°. To rise to 40° from 38° 53 20 at 146° 40 needs an increase of 1° 06 40 in arc and therefore of 3° 20 in longitude. Therefore the 40° zone starts (and the 30° zone ends) at 150°.

So far we have:

Up to 90° add 45°
From 90° to 150° add 30°
Past 150° add 40°.

This agree well with a fragment of ACT 821aa, which says: from 30° to 90° add 45°, from 90° to 150° add 30°. It also says: beyond 90° multiply by two-thirds.

From 170° to 233° 20 the arc increases by 1° for every increase of 2° in longitude, so the next added arc is 3/2 times 40°, i.e. 60°. To reach 60° from 58° 53 20 at 207° 46 40 needs an increase of 1° 60 40 in arc and therefore of 2° 13 20 in longitude. Therefore the 60° zone starts at 210°. After that the arc is still increasing at the same rate, so the next added arc is 90°.

We have so far at least five zones. Two of them, 90° to 150° and 150° to 210°, cover precisely two signs of the zodiac; ACT 811b groups the signs into pairs, including these two pairs. It is a reasonable guess that the pairs are the zones for Mars. If so, we have

From 30° to 90° add 45°
From 90° to 150° add 30°
From 150° to 210° add 40°
From 210° to 270° add 60°
From 270° to 330° add 90°
From 330° to 30° add x°.

We have only to calculate x and then check to see whether the resulting system agrees with the data in the tablets.

The arc starting at 270° ends at 352° 30. Adding 90° to 270° yields 330° plus 30°. Multiplying the 30° by x/90 gived us 330° plus x/3°, so x/3° = 22° 30, giving x = 67° 30. This does agree with the data. And the MF and EL in act 502 both use this system.

Mean periods

The relation between the mean synodic and sidereal periods can be found by noting when a synodic phenomenon repeats at the same point in the sky and using the fact (which is obvious from a heliocentric viewpoint and seems to have been known to the Babylonians) that if X synodic periods of an outer planet equal Y sidereal periods and take Z years, then Z = X + Y. At least, it is obvious for oppositions, which always take place at the same elongation, namely 180°, from the sun. The Babylonians seem to have assumed that this holds also for the other synodic phenomena. Strictly speaking, this gives the mean of the periods that occur between the observations, but for a reasonably long interval this mean will be fairly stable.

Two such relations are given in procedure tablets.
ACT 811a says: Mars 284 years 133 appearances 151 rotations.

That is: 133 synodic periods and 151 sidereal periods each take 284 years.
ACT 819 says: Saturn 9 rotations 265 years. So 9 sidereal periods equal 256 synodic periods (from the relation X + Y = Z).

The relation underlying a table using the zigzag system, if not found in a procedure tablet, can easily be deduced.

For example, in ACT 600 for the oppositions of Jupiter, each entry differs by 108 from the next, so a synodic period causes this much change. The total change from minimum back to minimum is easily found to be 1173. This corresponds to one revolution round the ecliptic and so takes one sidereal period. Therefore 1173 synodic periods equal 108 sidereal periods.

Such relations determine the synodic arcs. The mean distance covered by the planet round the ecliptic in one synodic period is Y/X revolutions and the synodic arc is the fractional part of this. So for Mars, where Y/X = 1 + 18/133, the mean synodic arc is 18/133 revolutions, which to the nearest minute is 48° 43', a figure that is actually given in ACT 811a.

Now let us look at the zonal system. The various added arcs represent the different speeds at which the phenomena progress through the zones.

If a particle covers distances a, b , c,... at speeds u, v, w,... the total time taken is a/u + b/v + c/w +... and the average speed is a + b + c + ... divided by this time. This is the weighted harmonic mean of the average speeds.

Therefore the mean synodic arc should be the weighted harmonic mean of the synodic arcs. This would be formidably difficult to achieve. Instead, the Babylonians arranged the added arcs to make their weighted harmonic mean equal to the mean synodic arc. For example, the weighted harmonic mean of the six added arcs for Mars in ACT 501 is 18/133. And for Jupiter both the two-zone system and two four-zone systems yield the same figure as the one deduced from the zigzag. However for Mercury we would have four different results: 848/2673 revolutions for MF, 480/1513 for EF, 388/1223 for ML, and 217/684 for EL.

I would like to thank John Britton and Dennis Duke for helpful comments on an early draft of this paper.

Reference

ACT: Astronomical cuneiform texts. Edited by Otto Neugebauer, 1955. (Tablets with numbers 800 or higher are procedure tablets).