Dating Star Catalogs and Globes

First, let's explain Schaefer's method to find the epoch of a star catalog. Rather than use a globe, let's assume we have an actual catalog, and we'll look at the added complication due to using a globe later. We start with the coordinates of each star in the catalog. Next, we try a series of dates, and on each date we compute the true position of each star. Comparing the true position of each star and the position in the catalog, we find the date that minimizes chi-sq, the sum of the squares of the differences (true – catalog) for all stars. This is, to a good approximation (which is nearly exact in ecliptical coordinates) the same as finding the date when the average difference (true-catalog), averaged over all stars, is zero. It is also, to the same good approximation (hence also exact in ecliptical coordinates), the same as finding the date of zero difference for each star, and then simply averaging those dates. So far, so good, and we now have an estimate of the epoch of the underlying catalog.

We now must estimate the statistical uncertainty in the date estimate. Schaefer does this by finding the dates on which the chi-square has increased by one unit from the minimum value. This is mathematically the same as first computing the standard deviation of the differences (true-catalog) for each star, and then using the fact that we have *lots* of stars, and that the standard deviation of the mean is given by the standard deviation of an individual star divided by $\sqrt{N}$, for N stars. So, for example, Schaefer gets the standard deviation of the mean as 55y using 72 stars, but if he used instead 1000 stars he would get 15y.

Let's illustrate Schaefer's method using a well-known example: the star catalog of the Almagest. The advantage of such a test case is that we can check whether the answer following from his method is useful. For the Almagest star catalog, the average error for each star is about 1/2 deg, but the position of the equinox is off by about 1 deg. A chi-square analysis of the Almagest star catalogue gives a mean date of about 53.8 A.D. ±1.5y. But since Ptolemy's stated epoch of the *Almagest* star catalog is 137 A.D., this is wrong by some 83 years, or more dramatically, by about 55 standard deviations!

So what went wrong? Schaefer's method is indeed estimating an epoch date of the catalog, and the statistical uncertainty in that date. But the date he is estimating is the *intrinsic* date of the catalog, which is not necessarily the date intended by the catalog's author. Indeed, let's suppose that, unknown to us, the person who first created the catalog had an intrinsic error in the underlying coordinate system, so that all longitudes were systematically off. This is almost inevitable of course – nobody in antiquity knew the position in the sky of the vernal equinox with absolute certainty – and in Ptolemy's case, his error was about 1.1 degree. This would obviously have the effect of shifting the date of *every star* by the same amount and in the same direction. It would then shift the estimated *intrinsic* date by the same amount, but it would have no effect whatsoever on the estimated standard deviation of individual stars, or on the estimated standard deviation of the mean intrinsic date. This means that Schaefer's estimated uncertainty in the date of the underlying catalog, which as we have seen can be made arbitrarily small by simply using more and more stars, does *not* include the effect of an error in equinox by the person who first created the catalog.

In addition, if the input data for our analysis comes not from a catalog, but from a globe, then there are additional problems. Let's imagine that someone wants to plot a lot of stars on a sphere. There are (at least) two ways to proceed:

1. draw a detailed coordinate grid on the sphere, with lines of constant coordinate at, say, 10 deg spacing. Then plot each star as best you can in the resulting grid.

2. (Ptolemy's instructions). start with a blank sphere, pick a pair of opposite points as poles, and draw the corresponding ecliptic. Next plot a reference star (e.g. Sirius) at an arbitrary longitude but the correct latitude (distance from the ecliptic). Now proceed to plot all remaining stars with respect to the reference star. At the end, draw a colure circle as accurately as you can. To do this, you will now need to use the measured RA of any one of your stars.

In both cases, the positions of all the stars on the globe must be in error by some amount larger than the scatter already in the catalog, and so the estimated standard deviation of the mean using a globe must be larger than that found using the catalog directly. Of course, in the practical cases considered by Schaefer – the Farnese globe and the lore in Aratus – no underlying catalog is available. In addition, if the person who drew the original globe used method two (almost certainly the case, for all practical purposes), the position of the equinox would be subject to an additional error if it was not drawn exactly (which, of course, would always be the case).

What about dating the Farnese globe? Schaefer's method, assuming it is applied correctly, gives us an estimate of the uncertainty in the derived *intrinsic* date of the underlying catalog. However, we must add to that an estimate of what might be a plausible error in the equinox of the underlying catalog. If the underlying catalog was really that of Hipparchus, then the error could be rather small – some fraction of a degree. We know this because it is possible to estimate the epoch of the five types of phenomena used in the *Commentary to Aratus*, and they have an average epoch of about 135 B.C. with a spread of about ±5 years. If the underlying catalog was that of Ptolemy, then the example given above applies. However, based only on evidence from the globe itself, the equinox could easily be in error by as much as Schaefer's estimate of the overall accuracy on the globe, some 4 deg or so, giving an unavoidable minimum uncertainty in the date of perhaps ±288 yrs. Compounding this with some uncertainty in plotting the equinox on the globe, plus the statistical scatter in star positions, a final uncertainty of perhaps ±300 years or more must be expected.