DATING THE ALMAGEST STAR CATALOGUE USING PROPER MOTIONS: A RECONSIDERATION

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When considering star catalogues, either modern or ancient, it is useful to distinguish between the date of the actual star position measurements, which might range over many months or even years, and the epoch of the catalogue, i.e. the date corresponding to the star positions quoted in the catalogue. In order to prepare a catalogue at a particular epoch, the catalogue's author must adjust the star coordinates from their measured values using formulae appropriate to the specific coordinate system of the catalogue. In the case of ecliptical coordinates, Ptolemy writes in the Almagest[1]

> "For each star (taken by constellation), we give…its position in longitude as derived from observation, for the beginning of the reign of Antoninus [= 137 July 20]…its distance from the ecliptic in latitude… The latitudinal distances will remain always unchanged, and the positions in longitude can provide a ready means of determining the [corresponding] longitude at other points in time, if we [calculate] the distance in degrees between the epoch and the time in question on the basis of a motion of 1º in 100 years, [and] subtract it from the epoch position for earlier times, but add it to the epoch position for later times."

The rate of precession quoted by Ptolemy is not accurate, but the general idea is approximately correct. Thus Ptolemy, and indeed all astronomers up to the late 1500's, believed that simply adjusting the ecliptical longitudes for precession while leaving the latitudes invariant would change the epoch of a catalogue. Tycho Brahe was the first to point out that the latitudes do indeed change with time due to the change in the obliquity of the ecliptic[2], and later Edmund Halley observed that both the latitudes and longitudes change due to the proper motion of the stars, i.e. the continuous change in relative position of the stars on the celestial sphere[3]. This opens the possibility, then, that by looking carefully at the quoted positions of stars in ancient catalogues, we might be able to learn something about the date of the actual star position measurements.

A recent article[4] in this journal by Dambis and Efremov uses the proper motions of stars to assign a Hipparchan epoch to the star catalogue in the Almagest. Using the 40 fastest moving stars in the catalogue, and considering together the ecliptical longitude and latitude motions, Dambis and Efremov conclude that the catalogue epoch is –89 ± 122 and so the hypothesis of a Ptolemaic epoch of +137 is rejected at a 94% confidence level. This conclusion remains even if 5-10 of the fastest moving stars are omitted from the analysis, although the statistical significance is diminished somewhat. Dambis and Efremov also discuss another method of dating using proper motions, which dates individual stars and then averages those dates, but this method yields inconclusive results.

The claim that proper motions of the stars during the approximately 265-year period between Hipparchus and Ptolemy constrains the epoch of the catalogue measurements is surprising. Table 1 shows the computed change in ecliptical latitude between the years –128 and +137 for the 20 fastest stars. The changes due to proper motion for ecliptical longitude are similar but are complicated due to precession and are not shown. We see that for only the very fastest stars do the changes in these coordinates exceed 10 arcmin, the typical binsize of the Almagest star catalogue. We shall also see that the coordinate changes even for the fastest stars are smaller than the estimated statistical uncertainties (about 23 arcmin for latitudes, see below) in the coordinate measurements in the ancient catalogue. Therefore the conclusion of Dambis and Efremov is problematical, and warrants a careful reexamination.

In order to compare ancient star catalogue data with the predictions for ancient epochs from modern measurements and precession theory, it is useful to have computer files for both the coordinates in the Almagest star catalogue and the modern coordinates, including proper motions, of those same stars. In my case, I began with an on-line copy[5] of the Almagest star catalogue derived from the edition of Manitius, and then I edited the coordinates and the star identifications in that catalogue to bring them into conformance with the more recent edition of Toomer[1]. For the modern star coordinates I have used a subset of the NASA SKY2000 catalogue[6] that corresponds to the Bright Star Catalogue[7] version 5. For the precession and proper motion calculations I have used the standard formulae given in Meeus[8]. Dambis and Efremov use similar modern data and the Almagest data from the book of Grasshoff[9]. I cannot be sure that we are all using exactly the same data, but it is very likely that any differences are not affecting our results significantly, since we are both averaging over large numbers of stars.

First, let us summarize the analysis of Dambis and Efremov. The basic idea of their 'bulk method' is to consider a group of N fast stars, to consider the observed differences between the catalogue value of the star coordinates and their values computed for an assumed epoch T_0 (they generally take $N = 40$ and $T_0 = 0$), and then to estimate and eliminate the systematic error component in that difference for each star coordinate, thereby isolating the random statistical measurement error. Thus, considering now just the latitudes β, Dambis and Efremov suppose that for each star in the catalogue

$$\beta_A = \beta_c(T) + \Delta\beta_{sys} + \varepsilon, \qquad (1)$$

where β_A is the value recorded in the Almagest star catalog, $\beta_c(T)$ is the value computed for the epoch T using modern star coordinates and precession theory, $\Delta\beta_{sys}$ is the systematic error in the ancient catalogue value, and ε is the random statistical measurement error in the ancient catalogue value. Further, since the proper motion of each star is linear in velocity μ_β (at least over the time spans we are considering), the time variation in β_c may be taken as

$$\beta_c(T) = \beta_c(T_0) + \mu_\beta (T - T_0). \qquad (2)$$

Note that the time dependence of β that results from the variation of the obliquity of the ecliptic is omitted from Eq. (2), but it will cancel in any event in Eq. (5) below. Therefore, defining $\Delta\beta(T) = \beta_A - \beta_c(T_0)$, for some reference epoch T_0, we have:

$$\Delta\beta(T) = \mu_\beta\,(T - T_0) + \Delta\beta_{sys} + \varepsilon \qquad (3)$$

where the random variable ε has mean zero and variance σ^2. In order to isolate the systematic error $\Delta\beta_{sys}$, we consider a number n of nearest neighbor stars of the target fast star (typically n is in the range 2 – 16) and we average Δβ over these stars, obtaining

$$\langle\Delta\beta\rangle_n = \langle\mu_\beta\rangle_n\,(T - T_0) + \Delta\beta_{sys} + \varepsilon' \qquad (4)$$

where the random variable ε′ has mean zero and variance σ^2/n due to the averaging. In fact, Dambis and Efremov compute the median of the n values rather than the mean, since the median is somewhat more robust but still has the 1/n effect in reducing the variance. I follow them in this choice, but I have checked it makes no significant difference in the final conclusions which choice is made. Note also that we assume that the systematic error term is the same for both the target star and each of the nearest neighbor stars, due to their locality on the celestial sphere. We then compute, for each star in the catalogue, the quantity

$$\Delta^2\beta = \Delta\beta - \langle\Delta\beta\rangle_n = (\mu_\beta - \langle\mu_\beta\rangle_n)\,(T - T_0) + \varepsilon'' \qquad (5)$$

where the random variable ε′′ has mean zero and variance $(1 + 1/n)\,\sigma^2$ (the 1/n term is actually more like 1.253/n if the median is used instead of the mean). In this way the systematic error term is largely removed at the cost of a small increase in the statistical error. Note also that, as mentioned above, the subtraction of nearest neighbors also eliminates the change in latitude due to the change with time of the obliquity of the ecliptic.

Since Equation (5) is of the form $y = bx$, ordinary least squares provides a determination of the slope $(T-T_0)$, using $(\mu_\beta - \langle\mu_\beta\rangle_n)$ as the *x*-values and $\Delta^2\beta$ as the *y*-values in the fit.

The entire procedure outlined above is also performed for the ecliptic longitudes λ, with the proviso that we always use the quantity $\lambda \cos\beta$ (and $\mu_\lambda \cos\beta$) in order to appropriately weight the errors in λ which naturally increase with latitude. Note that the subtraction of the systematic error in longitude also neatly cancels the changes due to precession. Least squares fits[10] are performed for the latitudes and longitudes alone, and with both coordinates together, assuming $N=40$, $n=6$, and $T_0=0$, with the results:

$T_\beta = -81 \pm 147$

$T_\lambda = -109 \pm 226$

$T_{\lambda\beta} = -89 \pm 122$

according to Dambis and Efremov, and

$T_\beta = -80 \pm 121$

$T_\lambda = -74 \pm 174$

$T_{\lambda\beta} = -77 \pm 102$

when I repeat their analysis as closely as I can (see Figure 1). The differences in the various values may be due to a number of factors, such as small differences in our catalogues or the treatment of outliers, but the differences are nowhere significant.

Now if we are to trust the conclusion of Dambis and Efremov that the data for both coordinates combined yield $T = -89 \pm 122$, and thus exclude a Ptolemaic epoch at a confidence level of about 94%, i.e. a 1.85σ effect, then we need to be sure that the statistical procedures used are as reliable as possible.

Dambis and Efremov are not explicit in explaining how they dealt with outliers, so I will explain what I did to (nearly) duplicate their results. First, consider all the catalogue stars and eliminate as potential neighbor candidates all those with errors more than three standard deviations from the mean of all the coordinate errors. This eliminates 19 stars in longitude and 18 stars in latitude from consideration as nearest neighbors. Among others, the longitude point for α Cen is omitted.

Second, when considering the data to be fit in Figure 1, I again impose a 3σ cutoff, but this time σ is computed on just the 80 stars in the fit. This cutoff eliminates the latitude points for α Cen and β CVn. Dambis and Efremov also eliminate the longitude and latitude points for θ Cen, so I follow them, although these points are only about 2.5σ from their means. This treatment of outliers is problematical, and will be discussed in detail below.

The method of Dambis and Efremov proceeds without using any direct information on the statistical errors in the values of $\Delta^2\beta$ and $\Delta^2\lambda \cos \beta$. They effectively use the scatter in the 40 or 80 points in their fits to estimate both the epoch of the measurements, i.e. the slope of the fit line, and the uncertainty in that slope. This method therefore both assumes and requires a good linear fit, but it is clear from Figure 1 that the data are not at all linear (the linear correlation is about –0.003, even with α Cen, θ Cen, and β CVn omitted) and the fit explains essentially none of the variance in the data. This is particularly critical in this case, because the method of Dambis and Efremov to estimate the uncertainty in the slope b, and hence the epoch T, effectively *assumes* a good quality of fit. The danger of proceeding under such a questionable assumption is a well-known hazard.[11]

If a linear fit is still required, though, then an alternative, and greatly preferable, procedure is to compute the estimated sample variances in $\Delta^2\beta$ and $\Delta^2\lambda \cos \beta$ for all the stars in the catalogue, excluding outliers, and to use these as the weights in the least squares fits. This does not overcome the objection that the data are not linearly correlated, but it does remove the need to use the scatter in 40 or 80 points to estimate the uncertainty in the slope. In this way we are then assuming only that the statistical error for the fast stars is the same as the statistical error for all the other stars in the catalogue. Of course, this assumption would have to be qualified if the fast stars happened by chance to be isolated into some corner of parameter space that was particularly well measured. For example, it could have been that the fast stars were accidentally clustered near some particularly well-measured constellation in the zodiac. However, I have checked all reasonable possibilities for such accidental bias, and it does not exist. The fast stars were measured just as well, no better and no worse, than the stars as a whole.

The full results for the systematic and statistical errors in latitude and longitude are shown in Figures 2-5. Figures 2-3 show the systematic errors in longitude and latitude for each star for the epoch +137, excluding only those potential neighbor stars whose original positional errors exceed the average catalogue error by more than three standard deviations. For simplicity I show $\langle\Delta\beta\rangle_n$ versus λ and $\langle\Delta\lambda \cos\beta\rangle_n$ versus λ for n = 6. These plots show the familiar systematic error in λ of about -1° (for the epoch +137), and also the (somewhat) periodic errors in λ and β first noticed by Peters.[12] Figures 4-5 show the values of $\Delta^2\beta$ versus λ and $\Delta^2\lambda \cos\beta$ versus λ, also for n = 6, and we see that in fact the systematic errors have been removed as promised. This removal is insensitive to the value assumed for n. The standard deviations σ_λ and σ_β for all the data in Figures 4-5 (1028 points in each chart) are 39 and 36 arcmin, respectively.

Now the standard deviations σ_λ and σ_β for the data in Figures 4-5 after removing 3σ outliers (19 in longitude and 18 in latitude) are 26 and 23 arcmin, respectively. Furthermore, these values vary only slowly for any value of n in the range 6 – 16. When I repeat the least-squares fits using Equation (5) and with these values as the standard deviations σ that determine the error in T (see the previous footnote), we obtain

$$T_\beta = -188 \pm 243$$

$$T_\lambda = -82 \pm 317$$

$$T_{\lambda\beta} = -142 \pm 195$$

for the fits to the latitude, longitude, and combined data, respectively. The reason that the errors are now so much larger than we found earlier is that the original errors were essentially controlled by the standard deviations of the values of $\Delta^2\lambda \cos\beta$ and $\Delta^2\beta$ for just the 80 fast stars, after a 3σ cutoff on just those 80 stars, and the elimination by hand of another two points for θ Cen as discussed above. These standard deviations are about 16 and 13 arcmin, respectively, compared to about 26 and 23 arcmin for the full catalogue sample. Thus we see that when the data are analyzed using the complete information on the errors, the uncertainties on the estimated parameters grow large. These fits clearly do not exclude either a Hipparchan or a Ptolemaic epoch for the star catalogue measurements.

It is highly questionable, however, whether the epochs implied by any of the fits have any value, since there is clearly no linear relation visible in the data shown in Figure 1, even if the 'outliers' of Dambis and Efremov are removed. If we consider each point in the figure as an (x_i, y_i) pair, then the slope (assuming a zero intercept) is given by $b = \sum_i x_i y_i \Big/ \sum_i x_i^2$. Now each point in the plot also gives an estimate of the slope (and hence the epoch), simply as $b_i = y_i / x_i$. These individual dates are then combined in a weighted average, $b = \sum_i w_i b_i \Big/ \sum_i w_i$, with individual weights $w_i = x_i^2$. Table 2 lists for each of the 40 fastest stars in longitude and latitude the coordinate pairs (x_i, y_i) and the dates implied, $60 \times y_i / x_i$. For any set of points, then, the fitted date is just the weighted average of the individual dates. It is clear that because of the absence of any linear relation in the data, the dates vary so widely and irregularly that no firm conclusion can be drawn from the average of the dates.

In summary, the analysis of Dambis and Efremov errs in substantially underestimating the random measurement errors in their data, and hence they get a smaller confidence interval on the epoch implied by their fits than is really justified. When the directly determined estimates of the random errors are used, the confidence interval determined is so large that the Hipparchan and Ptolemaic epochs cannot be distinguished. It is very likely, however, that the method proposed by Dambis and Efremov for separating the systematic and statistical errors in the star coordinate measurements will be useful in other investigations of the star catalogue.

The history of the star catalogue of the Almagest is long and complex, and over many hundreds of years numerous authors have debated when and by whom the star coordinates were measured. A fairly complete account of the history of the discussion up until about the mid-1980's is given by Grasshoff[13], and several commentaries since then can be found both in this journal[14] and in the journal DIO[15]. This paper, and the paper of Dambis and Efremov, looks only at using proper motions to try and distinguish the epoch of the star coordinate measurements. Other methods, such as looking at the pattern of fractional endings of the coordinates[16], the pattern of systematic errors in the catalogue[17], or the correlation between a star's magnitude and its inclusion or omission from the catalogue[18], are all independent and must be judged on their own merits. It appears doubtful, however, that considerations of proper motions can shed light on the issues.

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10. Given N pairs of data points (x_i,y_i), and defining $S_{xx}=\sum_{i=1}^{N} x_i^{\,2}$, $S_{yy}=\sum_{i=1}^{N} y_i^{\,2}$, and $S_{xy}=\sum_{i=1}^{N} x_i y_i$, the least squares fit of $y=bx$ yields $b=S_{xy}/S_{xx}$. If the standard deviation σ of the y's is known, then the estimated uncertainty of the slope b is $\sigma/\sqrt{S_{xx}}$. If the standard deviation of the y's is unknown, then it must be estimated from the fit and the error in b is instead $\sqrt{\dfrac{S_{xx}S_{yy}-S_{xy}^{\,2}}{(n-1)S_{xx}^{\,2}}}$.

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(a)	(b)	(c)	(d)	(e)	(f)
779	1325	-2.90	-10.99	-12.81	17%
110	5340	-2.27	-10.74	-10.03	7%
723	509	1.46	7.01	6.45	8%
527	5019	-1.39	-6.92	-6.14	11%
818	2491	-1.27	-3.51	-5.61	60%
247	6401	-1.18	-7.28	-5.21	28%
265	5933	-1.16	-6.72	-5.12	24%
848	2943	-1.14	-3.06	-5.04	65%
969	5459	-1.09	-6.65	-4.81	28%
180	219	-1.08	-3.53	-4.77	35%
261	6752	-0.96	-6.40	-4.24	34%
19	3775	-0.86	-2.02	-3.80	88%
125	6623	-0.76	-5.48	-3.36	39%
146	5914	0.76	2.17	3.36	55%
783	1136	0.74	4.82	3.27	32%
32	4375	-0.71	-2.18	-3.14	44%
279	6869	-0.68	-5.14	-3.00	42%
940	5288	-0.67	-4.30	-2.96	31%
814	2035	-0.65	-0.76	-2.87	278%
360	660	-0.63	-1.47	-2.78	89%

Table 1. Column (a) is the Bailey number of the star in the Almagest catalogue. Column (b) is the HR number from the Bright Star Catalogue. Column (c) is the velocity v_β (in arcsec/yr) in ecliptical latitude. Column (d) is the change in ecliptical latitude (in arcmin), computed from modern theory and due to all causes, over the time interval –128 to +137. Column (e) is the change in latitude estimating from multiplying the velocity in column (c) by the 265 year time interval. Column (f) is the absolute percentage difference between columns (d) and (e). This difference is due to the time dependence of the obliquity of the ecliptic and is not negligible, but it does largely cancel out when we use the method of Dambis and Efremov for eliminating the systematic errors. For comparison, the typical binsize in the Almagest catalogue is 10 arcmin, and the estimated statistical uncertainty in the Almagest measurements is about 23 arcmin (see text).

Longitude					Latitude				
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
969	5459	-3.53	108.63	-1846	779	1325	-2.89	11.51	-239
779	1325	-2.89	6	-125	110	5340	-2.26	-16.91	449
57	7462	-1.86	10.99	-355	723	509	1.56	-1.01	-39
723	509	-1.25	6.02	-289	527	5019	-1.33	8.57	-387
196	937	1.16	11.16	577	818	2491	-1.26	-19.12	910
360	660	0.93	5.94	383	247	6401	-1.11	17.32	-936
784	1084	-0.97	-6.77	419	265	5933	-1.15	11.18	-583
501	4540	0.77	-40.28	-3139	848	2943	-1.12	-20.98	1124
37	4785	-0.78	-33.28	2560	969	5459	-1.04	83.93	-4842
79	7957	0.7	27.24	2335	180	219	-1.06	-3.94	223
675	8852	0.68	4.21	371	261	6752	-0.95	22.81	-1441
19	3775	-0.6	26.05	-2605	19	3775	-0.79	12.25	-930
265	5933	0.65	4.45	411	125	6623	-0.77	9.76	-761
61	6927	-0.63	3.04	-290	146	5914	0.75	13.09	1047
425	2990	-0.56	13.37	-1433	783	1136	0.7	10.95	939
288	7557	0.58	-1.43	-148	32	4375	-0.67	21.39	-1916
503	4825	-0.55	-0.47	51	279	6869	-0.66	7.09	-645
527	5019	-0.56	-10.3	1104	940	5288	-0.64	-51.63	4840
557	6241	-0.57	-0.22	23	814	2035	-0.65	5.81	-536
279	6869	-0.6	7.91	-791	360	660	-0.5	-1.83	220
180	219	0.56	18.06	1935	678	8969	-0.45	-0.12	16
396	1656	0.52	4.53	523	326	8665	-0.49	3.93	-481
129	6212	-0.5	-2.01	241	196	937	-0.46	13.09	-1707
848	2943	-0.51	18.97	-2232	287	7602	-0.48	14.26	-1783
328	8697	0.45	-27.42	-3656	189	21	-0.46	-8.28	1080
921	4287	-0.39	-31.02	4772	794	1173	-0.46	-1.24	162
108	5185	-0.44	11.79	-1608	90	5404	-0.47	16.01	-2044
755	1543	0.46	22.28	2906	343	8961	-0.43	-14.41	2011
168	7949	0.46	-0.92	-120	222	1708	-0.42	-18.06	2580
818	2491	-0.42	-29.27	4181	413	1101	-0.37	35.48	-5754
506	4910	-0.34	-2.48	438	518	5338	-0.38	-6.41	1012
488	4534	-0.44	-12.61	1720	844	2040	0.41	21.73	3180
247	6401	-0.39	7.43	-1143	79	7957	0.4	29.21	4382
937	5168	-0.35	-20.86	3576	624	8322	-0.3	1.92	-384
83	8494	0.36	2.43	405	815	1983	-0.32	10.65	-1997
176	8130	0.36	-46.97	-7828	107	5235	-0.35	1.1	-189
933	4775	-0.29	10.2	-2110	20	3569	-0.27	9.43	-2096
537	5777	0.41	-6.94	-1016	557	6241	-0.3	0.41	-82
20	3569	-0.33	-2.68	487	57	7462	-0.29	-12.63	2613
467	4057	0.39	10.68	1643	937	5168	-0.25	-8.72	2093

Table 2. Columns (a) and (f) are the Bailey numbers of the star in the Almagest. Columns (b) and (g) are the HR numbers of the star. Columns (c) and (h) are the x-values (in arcsec/yr), and columns (d) and (i) are the y-values (in arcmin) in the x-y fits as discussed in the text. Columns (e) and (j) are the dates implied for each individual star, which are $60 \times y_i / x_i$.

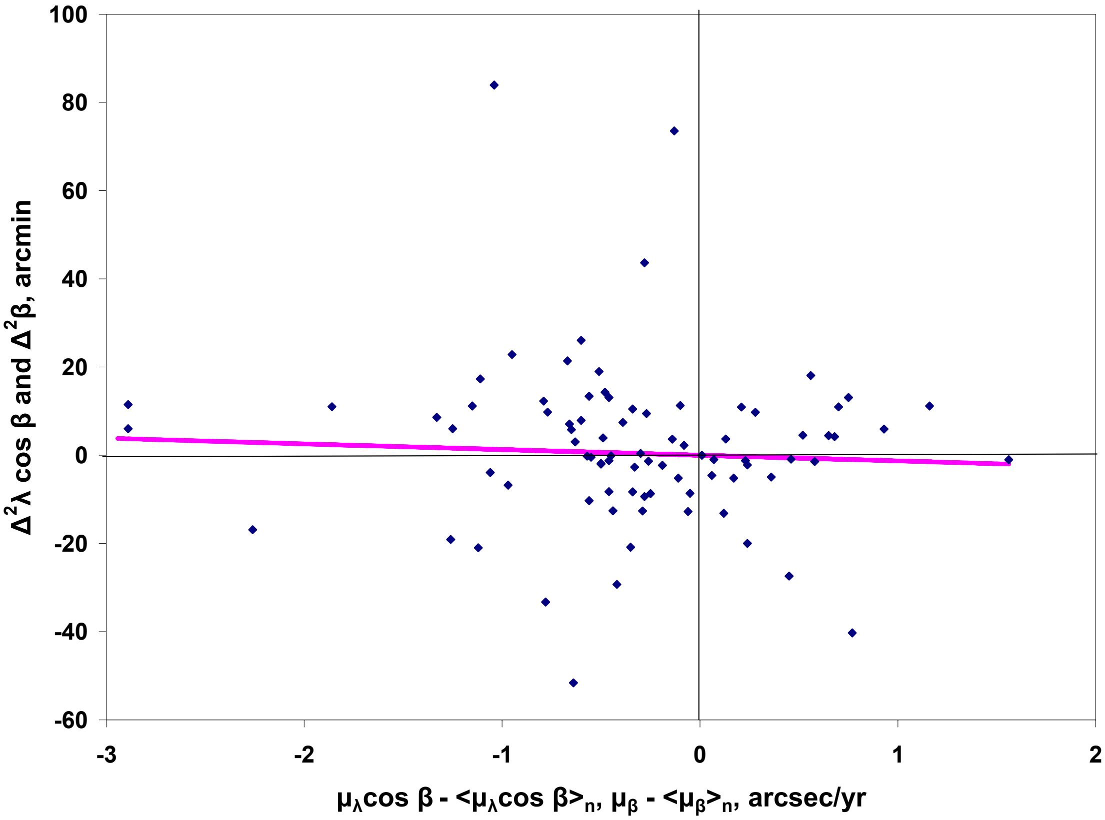

Figure 1. The analysis of Dambis and Efremov for the 80 fast stars in longitude and latitude. The three highest points on the chart are α Cen(β), β CVn(β), and θ Cen(λ), respectively. The lowest point is θ Cen(β). The linear correlation coefficient and the R^2 of the fit are both very nearly zero, even excluding the four ‘outliers’ noted above. None of the points is really an outlier with respect to the full set of data (see the dashed lines in Figures 4-5 and the discussion in the text).

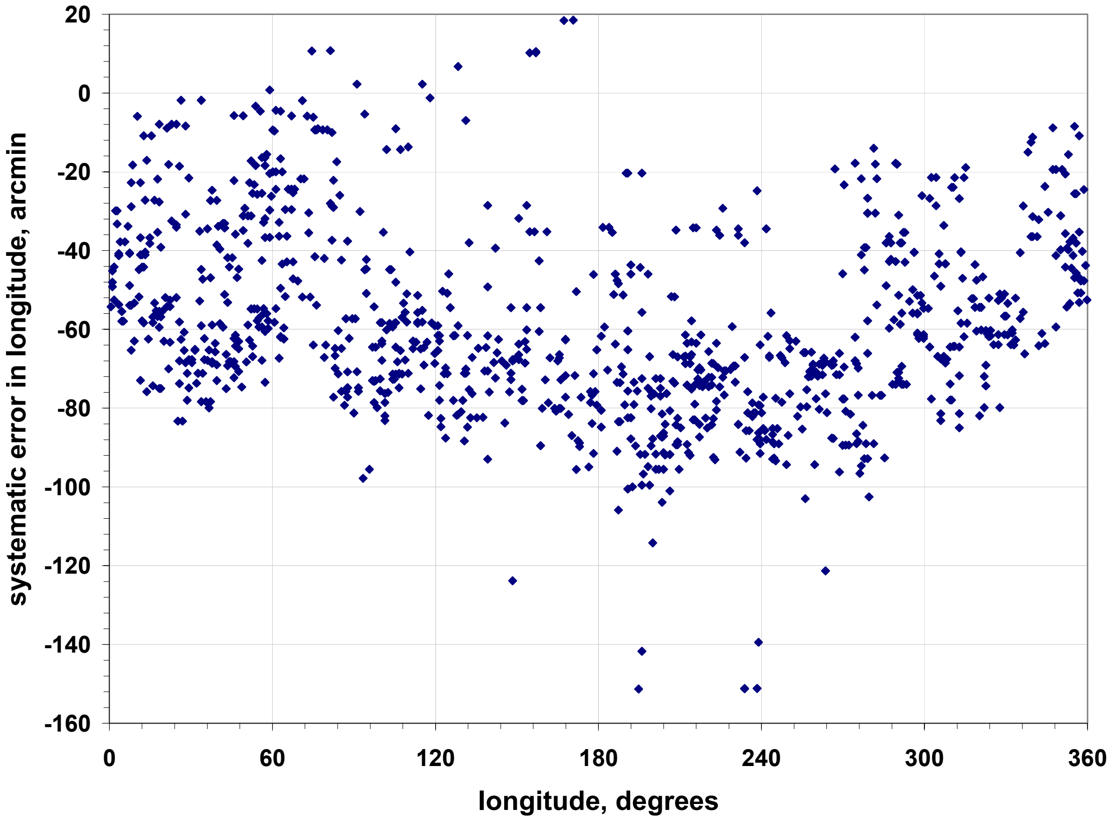

Figure 2. The estimated systematic errors in longitude for the 1028 Almagest stars. These systematic errors are estimated by averaging the errors of the six nearest neighbors to the target star.

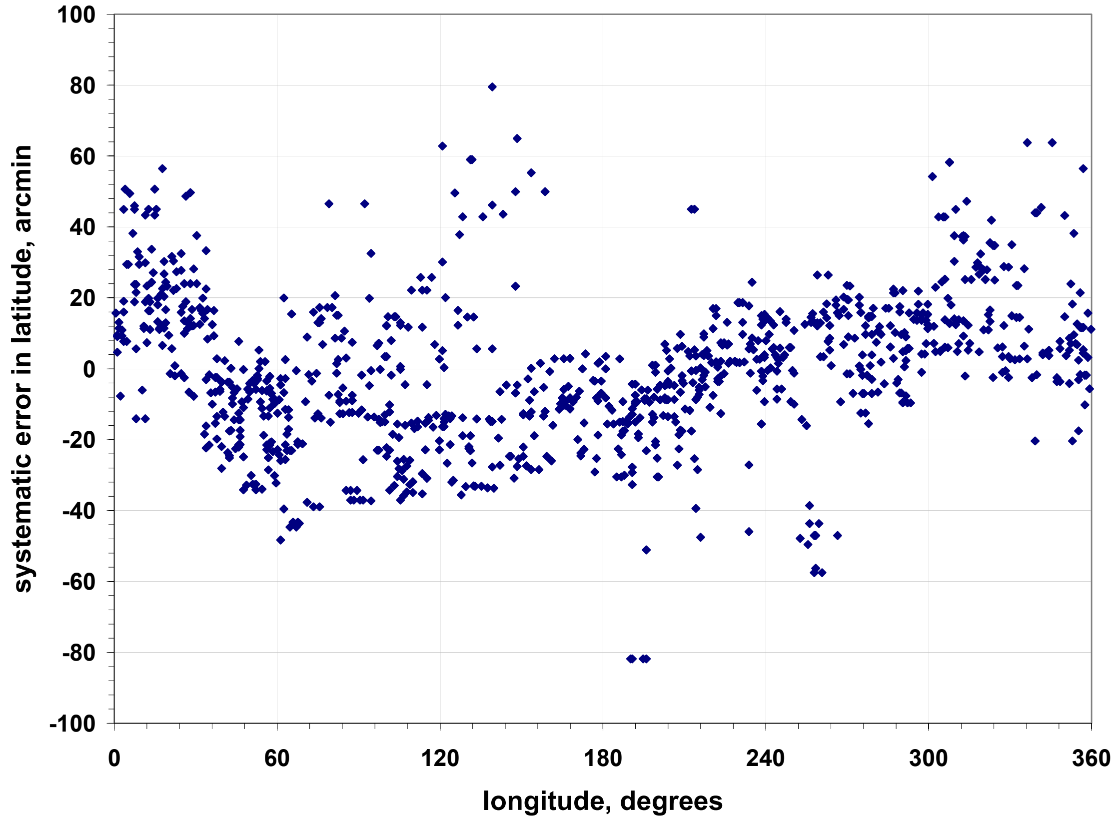

Figure 3. The estimated systematic errors in latitude for the 1028 Almagest stars. These systematic errors are estimated by averaging the errors of the six nearest neighbors to the target star.

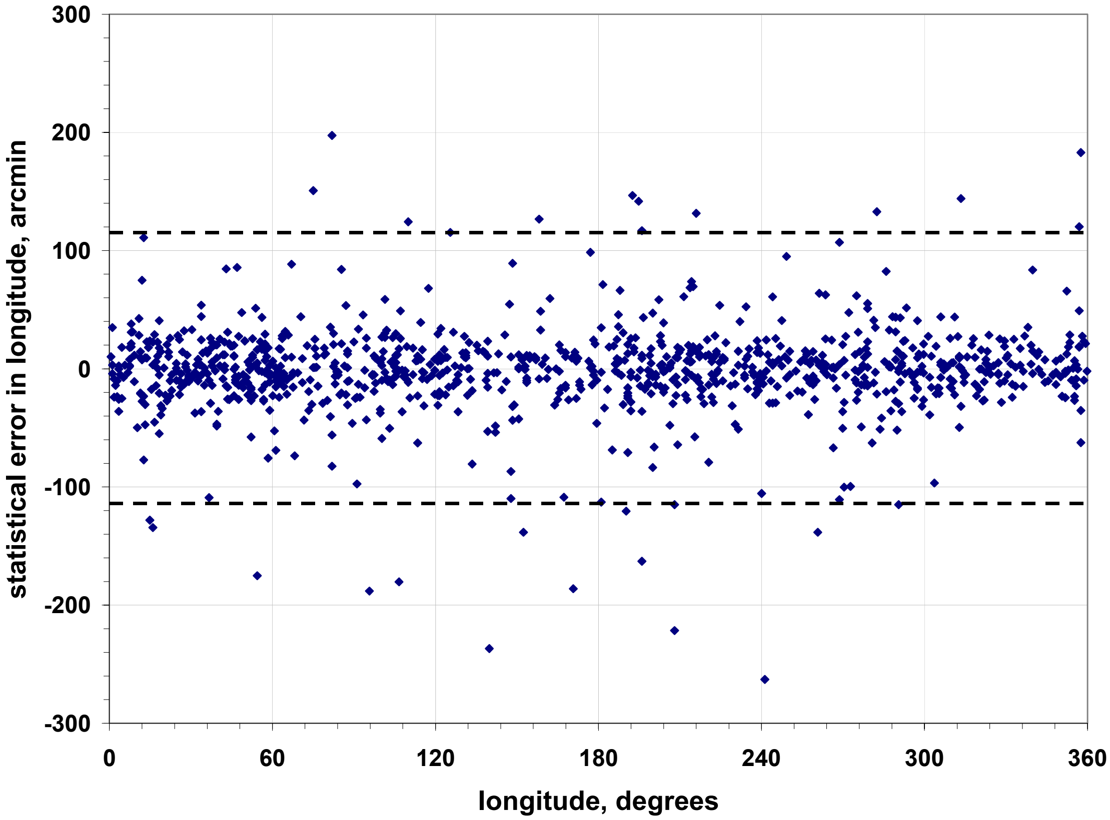

Figure 4. The distribution of estimated statistical errors in longitude for the 1028 Almagest stars. The standard deviation of all the stars is about 39 arcmin. The dashed lines show the 3σ cutoff points for outliers. The standard deviation for the stars after omitting outliers is about 26 arcmin.

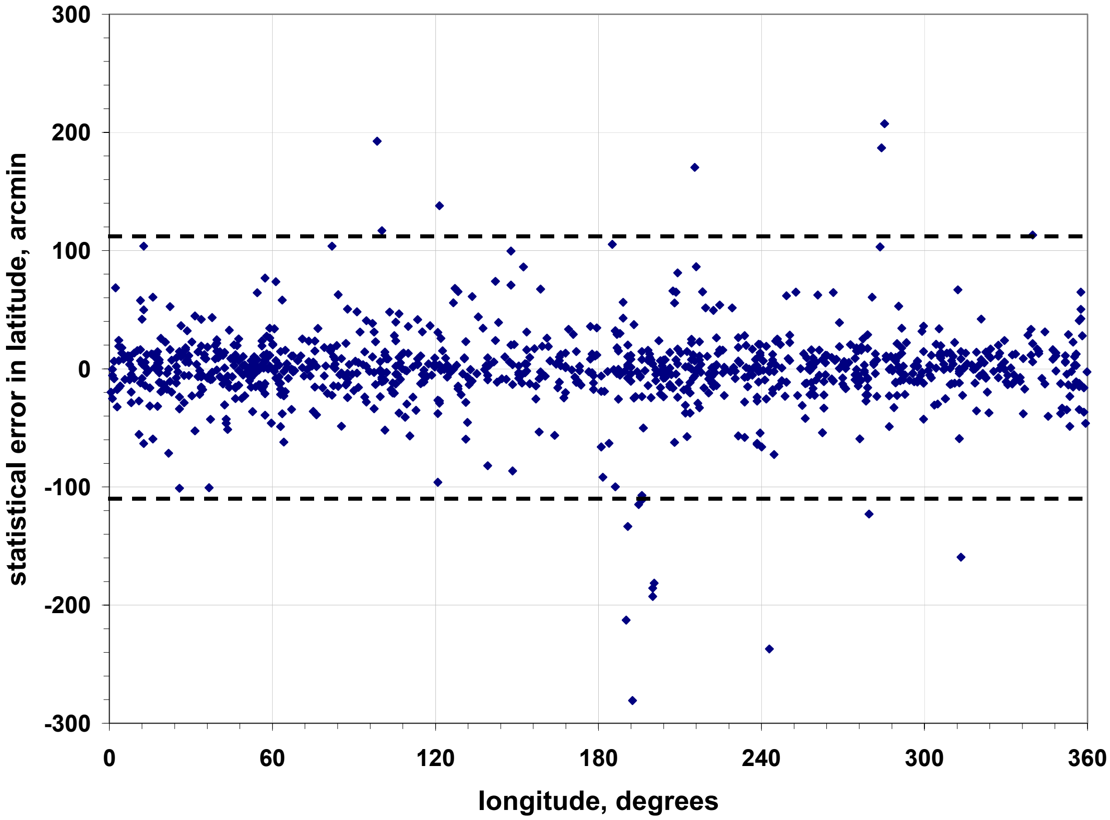

Figure 5. The distribution of estimated statistical errors in latitude for the 1028 Almagest stars. The standard deviation of all the stars is about 36 arcmin. The dashed lines show the 3σ cutoff points for outliers. The standard deviation for the stars after omitting outliers is about 23 arcmin.