Four Lost Episodes in Ancient Solar Theory

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The most widely known solar theory from antiquity is that of Hipparchus. All ancient sources are agreed that Hipparchus' model was a simple eccentric or an equivalent epicycle. The eccentric version of the model is shown in Figure 1. The Earth E is displaced a distance e from the center Z of the deferent. The purpose of the displacement is to allow the Sun at S to appear to move alternately slower and faster when seen from the Earth, even though it is at all times moving uniformly around the deferent as seen from the deferent center Z. Assuming a year length of 365¼ days and adjusting e and the direction of EZ with respect to the stars, one can account for the lengths of the seasons, which Hipparchus took as 94½ days for spring and 92½ days for summer, and it also follows that there are 88⅛ days in autumn and 90⅛ days in winter.

Figure 1

Geminus, writing about 50 B.C. and without explicit attribution to Hipparchus, gives the season lengths and describes how the varying speed of the Sun results from an eccentric deferent.[1] Theon of Smyrna, writing about A.D. 120 and following a presentation by Adrastus of Aphrodisias,

1) discusses the equivalence of eccentric and epicycle versions of the model;
2) gives the season lengths that are the empirical input data for the model;
3) gives a detailed diagram that is the basis for a geometrical derivation of the model parameters;
4) indicates the strategy of the geometrical solution; and
5) quotes the results: e/R = 1/24 and the direction of apogee is at Gemini 5½°.[2]

In a later passage about models structures in general, Theon does mention that Hipparchus remarked that the equivalence of the eccentric and epicycle models was worthy of attention, and that Hipparchus preferred the epicycle version.

Writing probably about one generation later than Theon, and about 265 years later than Hipparchus, Ptolemy gives in *Almagest* III the most detailed surviving account of the development of Hipparchus' model, including specific attribution to Hipparchus.[3] Ptolemy's discussion parallels Theon's in some ways. He repeats the same empirical input data, and like Theon does not mention how Hipparchus arrived at the season lengths he used. He uses a very similar geometrical figure and apparently a similar solution strategy, the principal difference being that Ptolemy supplies all the steps in the trigonometrical calculation.

Ptolemy's derivation of the model parameters is both simple and elegant. Given the season lengths and knowing the mean speed of the Sun from the length of the year, it is straightforward to calculate the lengths of the arcs TN and PK (see Figure 2), and once these are known the eccentricity $e = EZ$ and its direction relative to the vernal equinox follow immediately.

Figure 2

On the whole, Ptolemy's presentation is much more coherent and mathematically sophisticated than Theon's, but that might just be a consequence of different intended audiences, and so we really have no information about just how accomplished Adrastus might have been, compared say to Ptolemy.

In all of the surviving accounts what we are missing is any information on the actual development of the solar theory, with the result that there are numerous unanswered questions, e.g. how were the season lengths determined. This creates a noticeable shortcoming in what we would like to understand about the history of ancient mathematical astronomy. It seems therefore appropriate and useful to try and piece together what we can of the otherwise lost solar history of actual model development, basing our efforts on whatever attested material we can find, supplemented with informed and plausible speculation. At least for the four episodes discussed below there is enough attested source material to support the idea that each episode might have actually happened. Whether any of the episodes is ever confirmed depends largely on whether or not any new source material is discovered, an event that unfortunately does not happen often.

Lost Episode One: An Alternate Solution by Hipparchus

Just prior to giving his trigonometrical analysis of the solar model in *Almagest* III 4, Ptolemy writes

> "In order not to neglect this topic, but rather to display the theorem worked out according to our own numerical solution, we too shall solve the problem, for the eccentre, using the same observed data [94½ days and 92½ days]."

It is not entirely clear what Ptolemy is trying to tell us in this passage. He could be referring to earlier summaries such as Geminus' or Theon's, which, as discussed above, do not include numerical details, or he could be referring to the fact that Theon's approach differs in at least one numerical convention from his standard, namely that Theon divides the circle into 365¼ parts, clearly days, rather than the standard 360 degrees. But more likely Ptolemy could be saying that he is providing his solution because it is either different from that of Hipparchus, or he does not know the details of Hipparchus' solution, and so, presumably, would no one else in his time. For example, since Theon is only summarizing Adrastus, and possibly not doing that particularly well, there is no strong reason to suppose that Theon actually knew how Hipparchus solved the problem. Thus, in absence of further information, the actual method used by Hipparchus might be considered an open question.

Therefore, as our first 'lost solar episode', it seems of some interest to ask what type of solution Hipparchus might have used if he did not use the solution described by Theon and Ptolemy. Now it happens that we do have reliable source material that at the very least suggests an answer to that question. From Ptolemy's report in *Almagest* IV 11 we know with considerable certainty that Hipparchus knew and used a method for solving the general problem of determining the elements of an eccentric or epicycle model using three timed longitudes.[4] Here Hipparchus is seen applying this method to the Moon, using three longitudes associated with lunar eclipses, but clearly he could also use the same geometrical solution for the Sun when the three solar longitudes are at the cardinal points: the vernal and autumnal equinoxes and the summer solstice.

Figure 3

If he did solve the problem using the same geometrical steps, then his solution would proceed something like the following (see Figure 3). Let the vernal equinox, summer solstice, and autumn equinox be at T, K, Q, respectively. The deferent circle has center Z, radius *R,* and eccentricity *e* = EZ. Extend the line KE to B and let the distance *d* = EB set the scale of the diagram. The angles TZK, KZQ, and QZT are all known from the season lengths. Then the angles ETB and EQB may be computed, from which one determines QB, TB, and hence TQ, which gives the deferent radius *R* in terms of *d*. Angles QZB and TZB are then computed, and so KB is determined, whence one finds *R/e* and the angle TEZ (both independent of the value of *d*, of course).[5]

Naturally Hipparchus gets the same answers as always, since the problem is completely determined. This solution is undeniably less elegant than the solution given by Ptolemy, but there is really no question that Hipparchus knew how to solve the problem this way, and in the early days of trigonometry, the most elegant solution might not even be known. In addition, however, as we are about to see, there are definite advantages available to any analyst who did understand the analysis of more general longitude trios.

Lost Episode Two: Notebooks of Solar Position Data

So far we have been silently assuming that Hipparchus knew the dates of equinoxes and solstices, and those gave him the season lengths. We know from *Almagest* III 1 that Hipparchus knew the times of equinoxes and summer solstices to an average accuracy of about ¼ day (see Table 1).[6] Since no ancient source explains how these times were determined, our second 'lost solar episode' will consider just how an ancient astronomer would determine the time of an equinox or a solstice to that level of accuracy.

Table 1

By definition, an equinox occurs at the moment the Sun's declination is 0°, and a solstice occurs when the Sun's declination is at an annual maximum or minimum, i.e. $\pm\,\varepsilon$, where ε is the obliquity of the ecliptic, about 23;43° in Hipparchus' time. It is clear from multiple practical considerations that no one could have reliably and routinely simply noted the moment when the Sun's declination was at a given value: 0° for an equinox or ±23;43° for a solstice.[7] On the one hand, about half of the events will occur at night, when the Sun is of course not visible. On the other hand, even if the event happens in daylight, it is not always the case that the Sun will be unobscured by clouds and in a position in the sky favorable for measuring the declination accurately. Ptolemy's remarks in *Almagest* III 1 show that he was aware that equatorial rings could yield conflicting data, though it is not clear whether the multiple shadow crossings that he refers to were the result of distorted rings, as he believed, or an effect of refraction.[8] In addition, for the solstices it is impossible to achieve ¼ day accuracy with naked eye observations of any kind within a day or so of the event since the declination of the Sun is changing extremely slowly near a solstice.

It is most likely, then, that equinoxes and solstices were determined by observing noon solar altitudes for a series of days before and after the events. When the Sun is crossing the meridian at noon, it is relatively easy to measure its altitude h, and then knowing the geographical latitude φ, to compute the declination δ using

$$\delta = h + \varphi - 90^\circ .$$

But it is only at noon that such an easy determination is possible. Thus, one data collection strategy that was available at least from the time of Hipparchus and that would have worked well is the measurement of a daily series of solar altitudes at noon, when the Sun is crossing the meridian. A variety of instruments for making the observations could have been used, and I have verified that something as simple as a pierced gnomon, i.e. a rod with a pinhole at the top, gives h to more than adequate accuracy.

With a series of such measurements for a number of days before and after a cardinal event, it is fairly straightforward to use interpolation and the method of equal altitudes to estimate the time that the Sun's declination reaches some specific targeted value: 0° for an equinox, and maximum or minimum for a solstice. An example using simulated data for the vernal equinox in –134, which in reality occurred on March 24 at about 6:43 AM local time in Rhodes, is shown in Figure 4 and Table 2. Hipparchus reports that it occurred at midnight on March 23/24. From modern theory we can compute the true declination of the Sun at noon on the days before and after this equinox, and to simulate measurement errors we add to each declination a random error drawn from a normal distribution with mean zero and standard deviation ¼°. By simply looking at the trend of the declinations – they are monotonically increasing from negative to positive values – it is easy to estimate the approximate day of the equinox. An ancient analyst might then take pairs of declinations at equal intervals before and after the day of the equinox and estimate the hour of the event by linear interpolation, an arithmetic technique that we know was widely used in tabular calculations. Taking, say, eleven such estimates from intervals of ±2 days, ±4 days,…, ±22 days, the analyst would get eleven estimates of the hour, and the simplest final step would be to take the median of those values, as shown in Table 2.

Figure 4

Table 2

It is easy to estimate the average accuracy that results from such an algorithm by simulation of several hundred trial sets of data, and it turns out that if the standard deviation of a single measurement of the solar declination is ¼° then the standard deviation of a single median determination of the time of the equinox is about 4 hours. If the latitude φ is too large, then all the declinations will be too large by the same amount, and the times of vernal equinox will be too early, the times of autumn equinox too late. This is the pattern seen in the equinox times Ptolemy reports from Hipparchus (see Table 1). If the latitude is too small, the situation is reversed. As specifically noted by Ptolemy in *Almagest* III 1, an error of $\frac{1}{10}^{\circ}$ in declination in a single measurement leads to an error of ¼° in longitude or ¼ day in time, but as we have demonstrated with direct simulation, the use of repeated measurements implies that the errors in individual declination measurements could be 2½ times larger, or ¼°, and still allow the analyst to achieve a final error of only ¼ day in time.

During a solstice the declination of the Sun changes by less than 1° for about 17 days before and after the cardinal event, so the algorithm just described for equinoxes must be adapted. One simple adaptation is to measure solar declinations for some sequence such as 20, 25,…,50, 55 days before and after the solstice, and use that data to make multiple estimates of the times when the Sun is at equal altitudes, and hence declinations, before and after the solstice. The midpoints of such time intervals are then estimates of the time of the solstice, and as above a simple data reduction procedure would be to take the median value of the estimates. The standard deviation of the solstice time that results, if as above the standard deviation of a single measurement of the solar declination is ¼°, is found by simulation to be about 5 hours. The estimated solstice times are not affected by a systematic error in the declination due to a misspecified latitude, but they are fairly strongly affected by the changing speed of the sun over the weeks preceding and following the solstice.

We know from the comments of Ptolemy and his quotes of Hipparchus that calculation of some sort was used in conjunction with observations to determine Hipparchus' solstice and equinox times. Ptolemy first writes that in *On the displacement of the solsticial and equinoctial points* Hipparchus 'sets out those summer and winter solstices which he considers to have been observed accurately', and then quotes Hipparchus as follows: 'Now from the above observations it is clear that the differences in the year-lengths are very small indeed. However, in the case of the solstices, I have to admit that both I and Archimedes may have committed errors of up to a quarter of a day in our observations *and calculations* [emphasis added]'.[9] A few pages later Ptolemy writes 'For Hipparchus noted that in the thirty-second year of the Third Kallipic Cycle he had made a very accurate observation of the autumnal equinox, and says that he *calculated* [emphasis added] that it occurred at midnight, third-fourth epagomenal day.'[10] From these statements of Ptolemy and Hipparchus we can therefore be fairly sure that for both equinoxes and solstices series of daily altitude measurements were used to determine the time of cardinal events, even though no surviving ancient source has documented the details of such episodes. For all the equinox events that occur after dark or near the horizon, and for all the solstices, it is essentially the only viable option for achieving ¼ day accuracy, which, as discussed above, can be achieved with some persistence.

Beyond these solar measurements we can also mention the use of very similar techniques for the planets. For the inner planets Ptolemy cites some 14 instants of greatest elongation from the mean Sun for Mercury and 8 such events for Venus. However, to actually know that these are greatest elongations some analyst must have made a series of timed position measurements both before and after each event, just as discussed above for the equinoxes and solstices.[11] For the outer planets Ptolemy cites 9 oppositions from the mean Sun, 3 for each planet. Some of these actually occur during daylight hours, and for the same reasons as discussed above, all of them were certainly computed, as Ptolemy vaguely mentions in passing,[12] from a series of timed position measurements of each planet.

Thus we can be fairly sure that the method was used extensively by ancient Greek astronomers, starting no later than the time of Hipparchus, if not Archimedes. However, while we can be fairly certain that some ancient astronomer was determining equinox and solstice positions from sequences of observations, we must admit to being much less certain about the motivation. It is possible that the goal was, as Ptolemy clearly implies, a genuine desire for empirical determinations of the length of the tropical year, whether or not that length was constant, and of the season lengths for use as input to the determination of the solar orbit parameters. On the other hand, many people have suggested that Greek solar theory was strongly influenced by the transmission from Babylon of arithmetical schemes for luni–solar syzygy,[13] and that, for example, a major route of such transmission might have been a visit to Babylon by Hipparchus.[14] Regardless of how the transmission was achieved, it could well be that Greek astronomers had some level of *a priori* expectation for the season lengths as derived from the Babylonian models, and in such case the motivation for the solar observations could have been some mixture of trying to confirm their expectations as well as an empirical determination.

It might help to put the accuracy of the Greek measurements in perspective by comparing them to better documented but later results. By the ninth century A.D. several Arabic astronomers were measuring meridian solar altitudes with an average error of about 1 arcmin, near the resolution of the unaided eye.[15] Therefore it is not unreasonable to think that Greek astronomers working about one millennium earlier might have been able to make the same measurements with an average error of about 10–15 arcmin, and this is probably a very conservative estimate. This conclusion is supported by Ptolemy's reports in *Almagest* VII 3 of 18 stellar declinations measured by Timocharis and Aristyllos in about 290–260 B.C., and again by Hipparchus in about 130 B.C., that are on average accurate to about 10 arcmin.[16]

Lost Episode Three: Trio Solutions of Solar Position Data

We know that ancient astronomers determined the times of equinox and solstice many times. In *Almagest* III 1 Ptolemy reports 21 such determinations from Hipparchus alone – 6 autumnal equinoxes, 14 vernal equinoxes, and a summer solstice – and he implies that Hipparchus in fact determined many additional cardinal dates, especially for the solstices. Hipparchus himself says that Archimedes determined multiple solstice dates. It is clear from his *Commentary to Eudoxus and Aratus* that at least from the time of Hipparchus astronomers understood how, given the declination of any point on the ecliptic, to find the longitude of that point.[17] In modern notation the relationship is

$$\sin\lambda = \sin\delta / \sin\varepsilon \ .$$

Also, we know that this method of routinely determining solar longitude from noon altitudes is explicitly attested in many ancient Indian texts, all presumably of Greco-Roman origin and probably pre–Ptolemaic.[18] Altogether, this suggests that there might have been notebooks full of noon, and thus well-timed, solar altitudes and declinations for the days and weeks before and after the cardinal events, all of which could be easily converted to solar longitudes. And since noon altitude and declination measurements are so easy to make, solar longitudes might well have been determined on many, if not most, sunny days throughout the year.

Any trio of such well–timed longitudes would be appropriate input to a *general* trio analysis to determine the elements of the solar orbit, just as any trio of timed lunar eclipses can be analyzed to determine the lunar orbit elements. In fact, Ptolemy provides two such lunar analyses in *Almagest* IV 6, and the identical algorithm is used to determine the orbit elements of Mars, Jupiter and Saturn in *Almagest* X–XI (although here some additional iteration is needed to get the equant parameters).[19] Since we know that Hipparchus and his successors, and perhaps his predecessors, knew how to use the general trio method, and since there would likely have been an abundance of data as a by–product of determining the large number of cardinal dates, just counting the ones we have, then it seems possible, if not outright likely, that that data was used many times to directly determine solar model parameters, and so these multiple determinations constitute our third 'lost solar episode'.

These solar analyses would be no more or less complicated than the analysis outlined in the first episode above. The same algorithm applies, the only difference being that the angles BET and TEK are no longer 90°, but they are still accurately known.

Assuming that astronomers did make many solar longitude measurements and used these as input for many trio analyses, what would they find? We can estimate this by once again turning to simulation, and generating hundreds of trios of noon declination measurements which, when converted to solar longitudes by the method discussed in episode two, can be used to estimate solar orbit elements. Since there will be unavoidable statistical and systematic errors in these empirical values, the resulting solar orbit elements – the eccentricity e, the apsidal direction A, and the Sun's mean longitude $\bar{\lambda}$ at some reference time – will also show some scatter and systematic shift from the true values. In addition, since the assumed model is not really the correct one, essentially a Keplerian ellipse with a speed variation determined by the equal area in equal time law, there will also be some scatter in the determined values of e and A from the mismatch of the model and the data.

Thus, the analysts would have noticed that orbit elements seemed to vary from one analysis to the next, probably somewhat irregularly. Since there was no systematic understanding of statistical variation in measurement at that time, we cannot be sure how the analysts would have responded, although if the *Almagest* is a reliable guide, the response might well have been fairly pragmatic. In any event, the power of repeated measurement would have eventually asserted itself, and they would have noticed that while the Hipparchan value $A = 65½°$ is rather accurate, at least near Hipparchus' time, the value $e = 2;30$ is not at all accurate, and should instead be about 2;10. It is then conceivable that this variation might undermine their confidence in the solar model, and perhaps lead them to consider model complications beyond the single anomaly of the eccentricity (or epicycle). It therefore seems reasonable to suppose that one result of the experiences implied by this episode was the motivation and development of alternate models.

Lost Episode Four: The Concentric Equant

Although Hipparchus' eccentric/epicycle model is the best documented model from antiquity, it was not the only solar (or lunar) model used. The concentric equant model for the Sun is repeatedly attested in Indian texts, all of which are generally supposed to be of Greco-Roman origin, and the accurate value $e = 2;10$ is routinely used.[20] These concentric equant solar models therefore constitute our fourth 'lost episode'.

Figure 5

In the concentric equant model the Earth is at the center E of the deferent, but the center of uniform motion Z of the Sun at S is displaced some distance e from the center (see Figure 5). Even though the Sun is now always at the same distance R = ES from the Earth, the model still produces an apparent speed variation in the motion of the Sun such that in one direction (the direction EZ) the Sun seems to be moving slowest, and in the opposite direction it seems to be moving fastest. For the same season lengths used in Hipparchus' model, the concentric equant gives $e = 2;27,12$ and $A = 66;59,51^\circ$, virtually indistinguishable in practice from the exact results for the eccentric, $e = 2;28,55$ and $A = 65;25,43^\circ$.

Besides the mathematical details, Ptolemy's principal addition in *Almagest* III 3 to Theon of Smyrna's discussion of Hipparchus' solar model is that the eccentric model, and its equivalent epicycle version, is empirically justified by the fact that for the Sun the time required to move from least speed to mean speed is greater than the time required to move from mean speed to greatest speed. It is somewhat curious that Ptolemy would mention this so emphatically, when in fact it would have been impossible to verify for the Sun in any empirical sense using the measurement technology of his time. It could be, of course, that Ptolemy was mentioning the pattern of speed variation to emphasize a related fact, that the Sun appears to move slowest at greatest distance (apogee) and fastest at nearest distance (perigee). Within the class of normal epicycle models, this only happens when the Sun moves clockwise on the epicycle, for if the Sun moved counterclockwise, we would observe maximum speed at apogee and minimum at perigee, a simpler point that Theon had already made quite forcefully. So the fact that Ptolemy discusses speed variation in such detail could suggest that he was at least aware that other kinds of models can give a different pattern of speed variation. Using the concentric equant, for example, one finds that the time from least speed to mean speed is *equal* to the time from mean speed to greatest speed.

There is a clear reason that Ptolemy might have been aware of the concentric equant: he in fact uses a close variant of it to model the second lunar anomaly. His final lunar model is a concentric equant in that the center of uniform motion of the Moon is displaced from the center of the Moon's deferent, just as in Figure 4. The difference, however, is that in Figure 4 the Earth is at the center of the deferent, while in Ptolemy's final lunar model the Earth is at Z, the position of the equant.[21]

Final Remarks

There are, in addition, several other sources which clearly establish the existence of solar theories other than that of Hipparchus:

(1) There are clear indications in the *Almagest* that Hipparchus himself used solar models different from that attributed to him by Theon and Ptolemy. For example, the two pairs of eclipse longitude differences that Hipparchus uses to find the unusual lunar eccentricities in *Almagest* IV 11 may also be used to deduce the underlying solar models, and the resulting parameters are equally unusual: e = 7;48 and A = 76;25° for Trio A, and e = 3;11 and A = 46;09° for Trio B.[22] Although attempts have been made to understand the underlying models, the analyses are neither conclusive nor satisfying.[23] The solar parameters are so bizarre that we might be tempted to speculate that Hipparchus is somehow trying to use a lunar theory to learn something about the time variation of solar theory (the trios date to about –380 and –200), and so it is perhaps significant that in both trio analyses the eclipses all occur near equinoxes and solstices. The same proximity to cardinal events is true for the old Babylonian trio from about –720 that Ptolemy presents in *Almagest* IV 6, using data he probably also obtained from Hipparchan records.

(2) *Almagest* V 3 and V 5 give three timed solar longitudes due to Hipparchus, and these imply a solar model with parameters e = 2;16 and A = 69;05°, although it might be that the underlying model is actually based on season lengths of 94¼

days and 92½ days, for which the resulting parameters are instead e = 2;19 and A = 67;08° (as explained by Ptolemy in *Almagest* IV 11, the parameters deduced from trio analyses are very sensitive to small changes in the input data).[24] Note that either value of e is significantly improved over the 'standard' Hipparchan value 2;30.

(3) Theon of Smyrna mentions, quite routinely, a solar model with periods of 365¼ days in longitude, 365½ days in anomaly, and 365⅛ days in latitude. Whatever the empirical background of these numbers might be, the net progression of the fractions also suggests some degree of numerological tinkering. Theon also mentions that the Sun strays from the ecliptic by ±½°. Solar latitude was mentioned as early as Eudoxus, and must have had some level of use, since not only Theon but also Pliny mentions it, and Hipparchus felt compelled to deny its existence (although it is hardly clear from Theon that the model should or should not be associated with Hipparchus). Two papyrus fragments, *P. Oxy LXI.4174a* and PSI inv. 515, also known as PSI XV 1490, give solar motion tables that are clearly kinematic and are consistent with the model parameters mentioned by Theon, and so remove all doubt that the models mentioned by Theon were actively used.[25]

(4) *P. Oxy LXI.4163* is a fragment of a papyrus table from Oxyrhynchus that gives a template for daily longitudes of the Sun to degrees and minutes starting from the day of summer solstice, when the Sun is at Gemini 30°, i.e. Cancer 0°, and so the cardinal points are at the beginning of the signs. The recovered fragment covers only two months of motion so it is difficult to uncover the underlying solar theory, but all indications are that it is not based on the usual Hipparchan parameters. Indeed, it cannot be conclusively established that the model is even kinematical and not some variant of a Babylonian arithmetical scheme.[26]

(5) *P. Oxy LIX.4162* is similar to *P. Oxy LXI.4163* but appears to count days starting when the Sun is at perigee and puts the cardinal points at 8° of the signs. In this case the indications are strong that the underlying theory is kinematical, but even if it is, it seems not likely to be based on the usual Hipparchan parameters.[27]

(6) *P. Oxy LXI.4148* is a table of epoch values of summer solstices over a series of years. The dates are in error by about five days in the years covered in the fragment and are based on a year of length 365;15,22,46 days, which is almost certainly a sidereal year. Whether this table of epoch values was intended to be used with the template of *P. Oxy LXI.4163* cannot be established.[28]

In addition, there is reason to believe that complicated models of trepidation were developed, in which the position of the vernal equinox executes long period oscillations.[29] Altogether then, it is clear that throughout early Greek astronomy the development of a variety of solar theories was an active process.

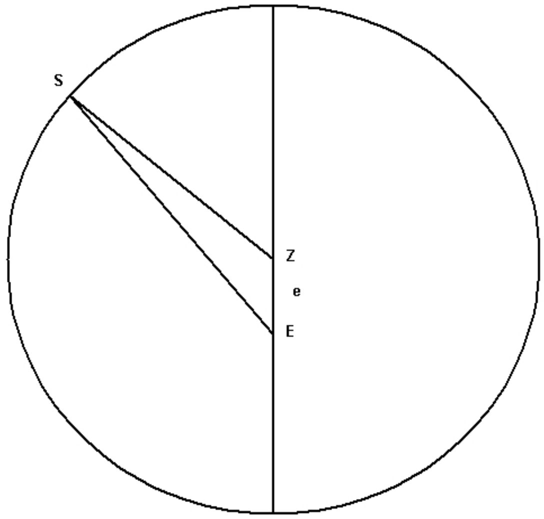

Figure 1

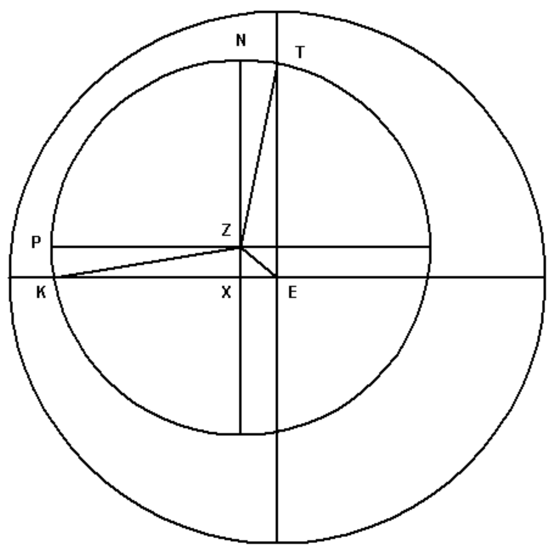

Figure 2

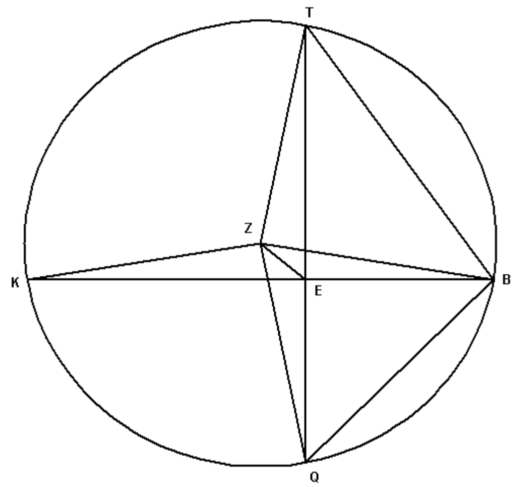

Figure 3

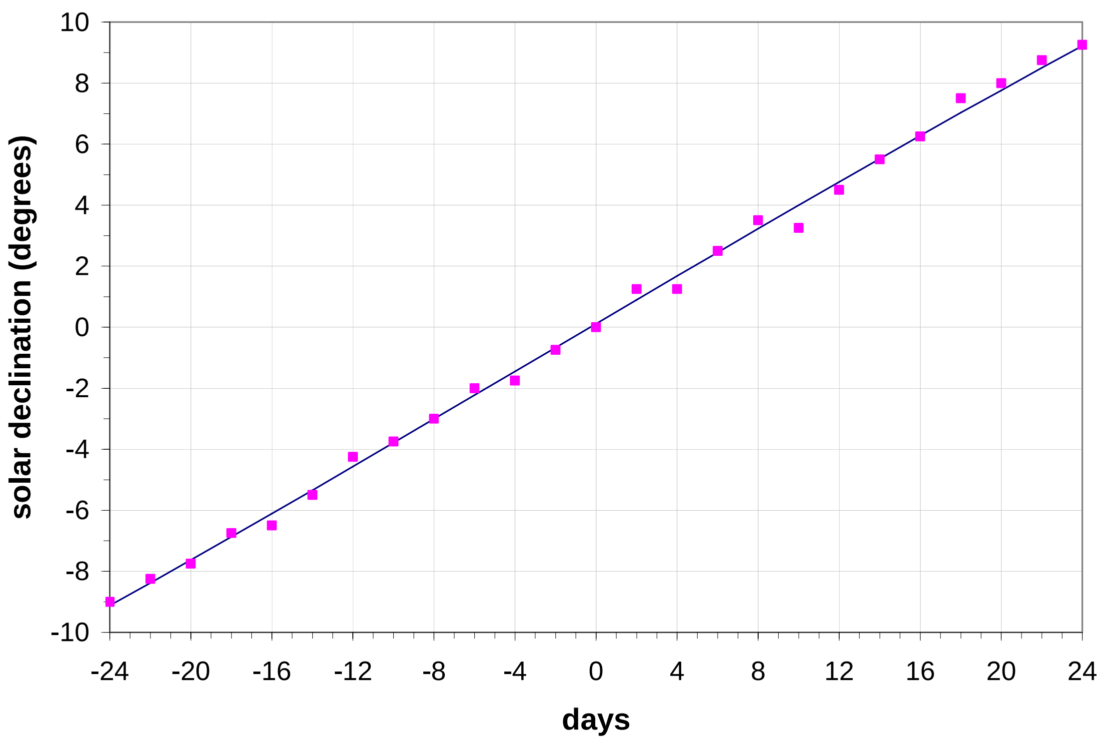

Figure 4. The boxes show simulated declinations of the Sun at local noon in Rhodes every second day for 24 days before and after noon of –134 Mar 24. The line shows the true declination of the Sun. The scatter in the data is due to assumed $\frac{1}{4}^\circ$ measurement errors in the declinations and rounding to the nearest $\frac{1}{4}^\circ$. The true vernal equinox occurred at about 6:43 AM.

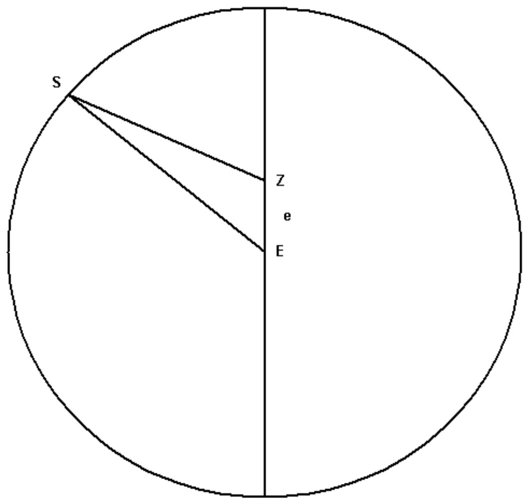

Figure 5

Accurate Dates			Hipparchus	Error (hrs)
-161	9	27.09	27.75	-15.8
-158	9	26.82	27.25	-10.3
-157	9	27.06	27.50	-10.6
-146	9	26.73	27.00	-6.5
-145	9	26.98	27.25	-6.5
-142	9	26.70	26.75	-1.2
-145	3	24.62	24.25	8.9
-144	3	23.86	23.50	8.6
-143	3	24.10	23.75	8.4
-142	3	24.34	24.00	8.2
-141	3	24.59	24.25	8.2
-140	3	23.83	23.50	7.9
-134	3	24.28	24.00	6.7
-133	3	24.53	24.25	6.7
-132	3	23.77	23.50	6.5
-131	3	24.01	23.75	6.2
-130	3	24.25	24.00	6.0
-129	3	24.49	24.25	5.8
-128	3	23.74	23.50	5.8
-127	3	23.98	23.75	5.5
-134	6	26.30	26.50	-4.8

Table 1. The times of Hipparchus' solstice and equinox determinations as reported by Ptolemy in *Almagest* III 1, and compared to the actual times from modern theory.

days before	declination	days after	declination	equinox time relative to noon
-2	-0.75	2	1.25	-0.50
-4	-1.75	4	1.25	0.67
-6	-2.00	6	2.50	-0.67
-8	-3.00	8	3.50	-0.62
-10	-3.75	10	3.25	0.71
-12	-4.25	12	4.50	-0.34
-14	-5.50	14	5.50	0.00
-16	-6.50	16	6.25	0.31
-18	-6.75	18	7.50	-0.95
-20	-7.75	20	8.00	-0.32
-22	-8.25	22	8.75	-0.65
			average	**-0.21**
			median	**-0.34**

Table 2. Column 5 gives the estimate of the time, relative to noon on day 0 (–134 Mar 24), that the declination of the Sun was 0°. The median puts the equinox at about 8 ¼ hours before noon, the average puts it at about 5 hours before noon.

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