Hipparchus' Coordinate System

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In his *Histoire de l'Astronomie Ancienne*[1] Delambre concludes unequivocally that Hipparchus knew and used a definite system of celestial spherical coordinates, namely the right ascension and declination system that we use today. The basis of Delambre's conclusion was disarmingly simple: he pointed out that in the *Commentary to Aratus*[2] Hipparchus actually quotes the positions of numerous stars directly in right ascension and declination (or more often its complement, polar distance)[3].

Nearly two centuries later, in his *A History of Ancient Mathematical Astronomy*[4], Neugebauer not only completely ignores Delambre's conclusion on this issue, but goes further to propose his own, as we shall see quite fanciful, theory that begins

> "From the *Commentary to Aratus*, it is quite obvious that at Hipparchus' time a definite system of spherical coordinates for stellar positions did not yet exist."

and concludes

> "…nowhere in Greek astronomy before the catalogue of stars in the *Almagest* is it attested that orthogonal spherical coordinates are used to determine stellar positions."

Today it is clear that Neugebauer's theory is conventionally accepted[5]. It is the purpose of this paper to offer fresh arguments that Delambre was correct.

Let us first review the facts as they appear in the *Commentary*. That work is divided into three sections. In the first, Hipparchus points out numerous quantitative problems with Aratus' qualitative descriptions of the stars and their relative positions. In the second, Hipparchus gives his own very specific data for the first and last stars to rise and set in each of 42 constellations, along with the degree of the ecliptic on the horizon and on the meridian at the moment when each of those stars is rising or setting. In addition, he lists by description other stars that are on or near the meridian at the time the first or last star in a constellation is rising or setting, and tells us the time required for the constellation to rise or set. Finally, in the third part, Hipparchus divides the celestial sphere into 24 equinoctial hours and tells us, beginning at the summer solstice, specific stars that are separated by one, or very close to one, equinoctial hour.

Now this third section is organized essentially according to right ascension as we know it today[6], and the hours quoted by Hipparchus are in fact very accurate indeed[7]. Hipparchus also tells us why he is giving us this information[8]:

> "This is useful for us both for determining with accuracy the hour of the night and for understanding the times of lunar eclipses and many other subjects contemplated in astronomy."

Further, in the first section of the *Commentary*, on every occasion that Hipparchus wants to tell us the position of a star, he tells us either the right ascension or declination of that star. To be sure he does not use the term right ascension, but rather 'circles parallel to the equatorial circle', writing[9]:

> "The star in the tip of the Bear's tail, which is the last of the seven toward the east, is located along a <circle> parallel to the equatorial circle at 4° of the Claws – I mean, when the tropic and equinoctial points are at the beginnings of the zodiacal signs."

And indeed, the star in the tip of the Bear's tail, η Ursae Majoris, had a right ascension in 130 BC of about 12h 17m, or 184.25°, which compares well with Hipparchus' quoted 184°. Hipparchus is here dividing the *equator* (and elsewhere circles parallel to the equator) into 30° segments and naming those with the usual twelve signs of the zodiac.

Thus the Claws (our Libra) is the 7th sign and begins at 180°. Hipparchus' habit is apparently not found in the work of any other ancient astronomer[10]. The first part of the *Commentary* contains numerous similar examples.

In Part 1 Hipparchus also often quotes the declination of a star, or equivalently its complement, the polar distance. Thus, when speaking of Cepheus[11]:

> "For the bright star in his right shoulder is 35 1/2° from the pole, and the bright star in the left shoulder is 34 1/4° away."

And indeed, the polar distances of α Cephei and ι Cephei were 35.55° and 34.33°, respectively, in 130 BC. And again, the first part of the *Commentary* contains numerous similar examples. Indeed, throughout the entire *Commentary*, on every occasion that Hipparchus wants to tell us the position of a star, he tells us either the right ascension or declination of that star.

Now let us come to part two of the *Commentary*. In the majority of the discussion in this part, Hipparchus quotes star positions, as elsewhere, in right ascension and declination. In the final sections, Hipparchus lists the first and last stars to rise and set in each constellation, the degree of the ecliptic on the horizon and on the meridian at the moment when each of those stars is rising or setting, a list of some stars that are on or near the meridian at the time the first or last star in a constellation is rising or setting, and the time required for the constellation to rise or set. A typical passage is[12]:

> "Bootes rises together with the zodiac from the beginning of the Maiden [Virgo] to the 27th degree of the Maiden. When it is rising, the section of the zodiac from the middle of the 27th degree of the Bull [Taurus] to the 27th degree of the Twins [Gemini] culminates. And the first star of Bootes to rise is the one on his head, the last is the one in the right foot.

Of other stars, on the meridian when Bootes begins to rise are Orion's left shoulder and left foot, both having gone about a half-cubit[13] beyond the meridian. When Bootes finishes rising, the bright star in the Dog's [Canis Major] haunches culminates.

All of Bootes rises in approximately 2 equinoctial hours."

Neugebauer elevated these passages into a theory of what he called "polar" coordinates[14], which he claimed were used by Hipparchus to establish stellar positions. One of his polar coordinates is ordinary declination, while the other is 'polar longitude', or *mediato coeli*. He defines this as the degree of the ecliptic culminating with a star. When the *Commentary* is read in context, however, it is plainly obvious that Hipparchus had no intention of associating the degrees of the ecliptic quoted in the final sections of part two with stellar positions. On the contrary, he explains at some length[15] that his purpose is to enable an observer to accurately tell the hour of the night by observing the risings and setting of constellations, and in the process correcting a number of mistakes in the passages of Aratus that also discuss this issue.

Beyond all the above, however, there are at least three additional lines of reasoning that tell us that Hipparchus could not possibly use, or even intend to use, the degrees of the ecliptic associated with each star as practical stellar coordinates.

First, it is obvious that the numbers quoted by Hipparchus as degrees of the ecliptic on the horizon at the same time a named star is on the horizon cannot possibly be the result of direct observation, since the stars on the horizon are almost always unobservable due to atmospheric extinction, and in addition the horizon is often otherwise obscured. The horizon numbers that Hipparchus quotes are thus far too accurate to be the result of naked eye observation. Polar longitudes are similarly unmeasurable, although for a different reason. The measurement of any longitude is always with respect to some other previously measured longitude (such as the Moon, and ultimately the Sun, since the origin, the first point of Aries, is invisible), and there is simply no way to measure one polar longitude with respect to another polar longitude, since the difference of two polar longitudes will not be a polar longitude. Therefore they cannot be considered *coordinates* for stellar bodies in any direct measurement sense[16].

Second, polar longitudes are in fact *never* quoted directly for a single star in the *Commentary*. Explicit examples have been previously interpreted as polar longitudes are seen in the passage above on Bootes: the polar longitudes of γ Orionis and β Orionis would be taken[17] as 56 1/2°. Indeed, Vogt[18] has used precisely these data, together with other data mentioned elsewhere for each star, to deduce each star's ecliptical longitude and latitude. However, it is clear that the number 56 1/2° is not associated with the simultaneously culminating stars γ Orionis and β Orionis but rather with the first star to rise, namely β Bootis (why Hipparchus is here ignoring the stars in Bootes' left shoulder, arm and hand: γ, λ, θ, ι, and κ Bootis, all of which clearly rise earlier, is a mystery[19]). In fact, the very positions of the stars that Hipparchus mentions in the second paragraph do not show up in the numbers Vogt and Neugebauer presumed as polar longitudes in any way whatsoever because the coordinates of those stars are not used by Hipparchus to compute the quoted value of the *mediato coeli*, which as we have seen is whatever he quoted for the degree of the ecliptic culminating when some *other* star is rising (or setting, as the case may be), and those values are associated directly with the stars mentioned in the first paragraph . In addition, the stars mentioned in the second paragraph are likely to be only near the meridian, and also distributed on both sides of it. In fact, in many cases, including our example of Bootes, Hipparchus states explicitly that the stars are a half-cubit or so before or after the meridian. As an interesting corollary, this means that Vogt's analysis of the correlations between his derived coordinate errors and the Almagest star catalogue coordinate errors is literally meaningless for those stars for which he used the polar longitude as input, and this is the majority of his cases. Further, as Grasshoff[20] and I[21] have discussed elsewhere, Vogt's conclusions can be largely dismissed on a number of other grounds.

Third and finally, since Hipparchus did not measure the rising, setting, and culmination numbers directly in the sky, he must have computed the numbers somehow, using some *other* set of numbers as input to the calculation. The closest he comes to giving us a worked example[22] is the case of υ Bootis, for which he tells us directly the declination and right ascension and then leads us step by step to the degree of the equator culminating, the degree of the ecliptic culminating, and the degree of the ecliptic rising. If he knew enough spherical trigonometry, he could compute these numbers directly. Whether he knew enough spherical trigonometry to do this is not known[23], but we can get some idea by looking directly at his quoted pairs of (rising, culminating) and (setting, culminating) values as they appear in part two of the *Commentary* and asking if they look like they came from direct computation. This is complicated somewhat by the fact that Hipparchus quotes values in units of at least a half-degree, so they are very likely rounded off somehow. Of the 168 pairs, 32, or about 20%, of the pairs have computed values of the culminations that differ from his quoted values by more than 0.7°. This might be a little too large a fraction if the values are hand-computed.

The alternative is analog computation on his celestial globe[24]. Certainly the observed rounding of numbers in the *Commentary* would be a natural result of such a procedure. Now to actually *plot* star positions on a globe, to the degree of accuracy implied by the data in the Commentary (about one degree), you obviously have to use *some* sort of 'definite system of spherical coordinates', which Neugebauer assured us 'did not yet exist' at the time of Hipparchus. Since, as outlined above, there is overwhelming explicit evidence in the Commentary that Hipparchus definitely had at his disposal, and clearly understood how to use, ordinary equatorial coordinates, the obvious conjecture is that he plotted star positions on his globe using these equatorial coordinates. Then he would get the numbers in the first paragraph by simply rotating his sphere until the star of his choice was rising or setting, and read off the degrees of the ecliptic that are on the horizon and the meridian. This would give him each of the numbers he needs for the first paragraph for each constellation. He also has to now look at the meridian, and see what stars are near it (in the case of Bootes, he finds three such stars). But as we discussed above, whatever coordinates he used to place these second paragraph stars on his globe are not reflected in the first paragraph numbers, and more important, those numbers should not be used to infer anything at all about exactly what coordinates Hipparchus actually used to plot those stars on his globe. So either way – direct calculation or using a globe – Hipparchus must have used as input some *other* set of stellar coordinates. The extensive use of right ascension and declination throughout the rest of the *Commentary* suggests strongly that those are the set he used.

Finally, there are two points of possible incidental interest. First, Ptolemy tells us explicitly in *Almagest* VII.1 that Hipparchus had a globe. Further, in *Almagest* VII.4 Ptolemy tells us why he himself is giving us his table of star positions, writing "In order to display the arrangement of stars on the solid globe according to the above method, we have set it out below in the form of a table in four sections." Thus, at least in Ptolemy's case, the very purpose of a star catalog was to help create a celestial globe.

Second, in *Almagest* VII.4 Ptolemy goes to some pains to explain to us that his use of 'to the rear of' and 'in advance of' and 'to the north of' and 'to the south of' refer directly to ecliptical coordinates. However, there are several cases where his star descriptions use this terminology but are not in accord with the facts. Toomer points out two examples of this.[25] What is interesting is that in each case the wording *is accurate* in equatorial coordinates. In addition, in the *Commentary* Hipparchus frequently uses the same terms 'in advance', 'in the rear', 'north', and 'south', and they are always accurate with respect to equatorial coordinates, but not always with respect to ecliptical coordinates. So it is plausible that Ptolemy copied the star descriptions he used from some Hipparchan document that was accurate in equatorial coordinates, but occasionally forgot to change them to be uniformly accurate not for equatorial coordinates but for ecliptical coordinates.

The above discussion leaves open the question of who decided when to convert star coordinates to ecliptical. Regarding potential star identification problems in his catalogue, Ptolemy tells us directly that[26]

> "one has a ready means of identifying those stars which are described differently [by others]; this can be done immediately simply by comparing the recorded positions."

thereby implying that he was not the first to use ecliptical coordinates in a star catalogue[27]. One obvious possibility is that Hipparchus switched to ecliptical coordinates after he discovered precession, but at this point that is just speculation, with little or no real evidence to back it.

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REFERENCES

[1] J. B. J. Delambre, *Histoire de l'Astronomie Ancienne,* (1817, reprinted New York, 1965), v. 1, p. 117, 172, 184.

[2] Hipparchus, *Commentary on the Phenomena of Aratus and Eudoxus*, trans. Roger T. Macfarlane (private communication). Until this is published, the interested reader must use Hipparchus, *In Arati et Eudoxi phaenomena commentariorium*, ed. and transl. by K. Manitius (Leipzig, 1894), which has an edited Greek text and an accompanying German translation.

[3] J. B. J. Delambre, *op. cit.* (ref. 1), p. 187, which lists the declinations and/or right ascensions from Hipparchus for 119 stars.

[4] O. Neugebauer, *A history of ancient mathematical astronomy*, (3 vols., Berlin, 1975), p. 277-80.

[5] See, for example, G. J. Toomer, *Hipparchus*, Dictionary of Scientific Biography 15 (1978), p. 217; J. Evans, *The History and Practice of Ancient Astronomy*, (New York, 1998), p. 103; G. Grasshoff, "Normal star observations in late Babylonian astronomical diaries", *Ancient astronomy and Celestial Divination* (1999), ed. N. Swerdlow, p 127 and footnote 23.

[6] The main difference being that Hipparchus does not define a zero point of right ascension, perhaps because he never needs one, but instead gives the six hourly intervals between the solsticial and equinoctial colures.

[7] H. C. F. C. Schjellerup, *Recherches sur l'astronomie des anciens. I. Sur le chronomètre Céleste d'Hipparque,* Urania 1 (1881) p 25-39.

[8] Hipparchus, *op. cit.* (ref. 2), section 3.5.1.

[9] Hipparchus, *op. cit.* (ref. 2), section 1.5.10-11.

[10] I am grateful to David Dicks and Alexander Jones for assurance on this point.

[11] Hipparchus, *op. cit.* (ref. 2), section 1.7.20.

[12] Hipparchus, *op. cit.* (ref. 2), section 2.5.1.

[13]In normal Hellenistic usage, the cubit is 2° (see, e.g. *Ptolemy's Almagest*, transl. by G. J. Toomer (London, 1984), p. 322 n. 5). Analysis of 45 stars for which Hipparchus uses

the cubit shows that his cubit is about 1.94 $\pm$ 0.15 degrees. Grasshoff's analysis, *op. cit.* (ref. 5), p. 137, shows that the Babylonian cubit is about 2.4 $\pm$ 0.1 degrees.

[14] Neugebauer, *op. cit.* (ref. 4), p. 1081.

[15] Hipparchus, *op. cit.* (ref. 2), section 2.1.1-2.1.14.

[16] One could, of course, imagine using a zodiacal armillary sphere or some similar instrument, aligning it to a star, and then reading off the polar longitude. This requires, however, the use of some *other* longitudinal coordinate (either ecliptic or equatorial would do), thereby rendering the polar longitude a derivative measurement, and not a primary coordinate.

[17] Assuming with Manitius, Vogt, Neugebauer (e.g. *op. cit.* (ref. 4) p. 278–9) and Grasshoff that when Hipparchus refers, e.g. to the *second* degree of a sign, we refer to that as the *first* degree of a sign. Similarly Hipparchus call the 30th degree of a sign what we call the 29th or last degree, and what we call the 0th or first degree he calls the beginning of the sign. Note, though, that Hipparchus uses this convention only when he refers to degrees of the ecliptic. When he refers to degrees of equatorial or declination circles, he uses our usual convention.

[18] H. Vogt, "Versuch einer Wiederstellung von Hipparchs Fixsternverzeichnis", *Astronomische Nachrichten*, 224 (1925), cols 2-54.

[19] Paul Mills (private communication, 2002). Elsewhere in the *Commentary* (section 2.2.14) Hipparchus explicitly mentions the stars in the left elbow and hand of Bootes, so he is not using a smaller constellation that we usually assume.

[20] G. Grasshoff, *The history of Ptolemy's star catalogue* (New York, 1990), 99-121.

[21] D. Duke, "Associations between the ancient star catalogues", *Archive for history of exact sciences,* (forthcoming).

[22] Hipparchus, *op. cit.* (ref. 2), section 2.2.25-29.

[23] Several people have suggested that Hipparchus could have used an analemma in these calculations: Neugebauer, *op. cit.* (ref. 4), p. 301; Toomer, *op. cit.* (ref. 5), p. 210; C. Wilson, "Hipparchus and Spherical trigonometry", *DIO* 7.1 (1997) 14-15; D. Rawlins, "Hipparchos at Lindos: a modest confirmation", *DIO* 7.1 (1997) 16-17.

[24] R. Nadal and J.-P. Brunet, "Le Commentaire d'Hipparque I. La sphère mobile", *Archive for history of exact sciences*, 29 (1984), 201-36 and "Le Commentaire d'Hipparque II. Position de 78 étoiles", *Archive for history of exact sciences*, 40 (1989), 305-54.

[25] Toomer, *op. cit.* (ref. 13), in n. 110 on p. 344 and n. 120 on p. 347. Toomer also includes another case in n. 117 on p. 346, but his discussion is in error in that footnote.

[26] *Almagest, op. cit.* (ref. 13), p 340.

[27] And further, since he says the comparison may be done 'immediately', Ptolemy is perhaps also telling us that other star catalogues in ecliptical coordinates were readily available, both for his readers and for himself (Noel Swerdlow, private communication, 2001).