On the Clarity of Visibility Tests

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The Almagest[1] star catalog (ASC) has for centuries invited speculation about who actually compiled it. Of its many curious features, the fact that the catalog contains no stars which are visible in Alexandria but not visible in Rhodes suggested to Delambre[2] that perhaps the catalog was actually compiled by Hipparchus, who is known to have lived in Rhodes (at about 36° north latitude), and not by Ptolemy, the author of the Almagest, who is known to have lived in Alexandria (at about 31° north latitude).

In 1982 Rawlins[3] constructed a model that, subject to its assumptions, provides a quantitative test of how well the catalog's southern limit tells us the latitude of the observer. Rawlins' application of the model produced a clear signal in favor of an observer at the latitude of Rhodes. In 2001 Schaefer[4] used Rawlins' basic model but with an updated set of technical inputs to reach a substantially different conclusion, basically favoring an observer at the latitude of Alexandria for at least three quadrants of the sky. It is the purpose of this paper to carefully examine exactly how the model works, and how conclusive are the results of either Rawlins' or Schaefer's analysis.

Here is how the model works: we assume as input all the stars in the sky that are visible to the naked eye, and a catalog, in this case the ASC, that contains some subset of these stars. For each such star, we compute its apparent magnitude m and a probability of visual detection P_{det}, which is a function of m. Then the probability that the ith star is included in the catalog is $P_i = P_{\text{det}}$, while the probability that a given star is not included is $P_i = 1 - P_{\text{det}}$. Finally, we form the likelihood, $L = \prod_{i=1}^{N} P_i$, which gives us a quantitative measure that the catalog in question was actually assembled subject to our assumptions.

The details of the calculation include the computation of the apparent magnitude m and the probability of detection P_{det}. The apparent magnitude is determined by adjusting the tabulated visual magnitude V for atmospheric extinction of the star's visible light. Briefly, we assume that a star is actually observed at its meridian culmination height $h = 90° - \varphi + \delta$, where φ is the latitude of the observer and δ is the declination of the star. The epoch T of the observer also matters, as the star's declination is affected by precession. The height h determines the depth X of the Earth's atmosphere that the star's light traverses, and given an extinction coefficient k, the apparent magnitude is given by $m = V + kX$. In practice[5], the depth X is usually split into components for Rayleigh scattering, ozone absorption, and aerosol scattering, each with its own extinction coefficient. Given m, the calculation proceeds with the computation of P_{det}. In general, we expect P_{det} to be near unity for bright stars and near zero for very dim stars. Rawlins used a piecewise monotonic function for P_{det}, while Schaefer used a specific functional form $P_{\text{det}} = 1/(1 + e^{F(m-m_0)})$, which introduces two parameters F and m_0.

In order to compute the likelihood L we must know values for the parameters φ, T, and k, and in Schaefer's version of the model, also F and m_0. We use a computer program to vary the parameters until the likelihood L is maximized, or equivalently, until the log-likelihood $S = -2\sum_{i=1}^{N} \ln P_i$, is minimized. Thus the model assigns penalty points (values of S) to an observer who either includes in his catalog a dim star or omits a bright star. Complete details are given in the papers of Rawlins and Schaefer.

First, we summarize Rawlins' analysis. He chose for the input sample of stars not all visible stars in the sky, but instead a subset of 30 southern stars that are in a sparsely populated area of the sky. Of these, 16 are included in the ASC, 14 are not. Assuming essentially zero scattering by atmospheric aerosols, Rawlins found $S = 14.4$ for Hipparchus' latitude and epoch and $S = 75$ for Ptolemy's latitude and epoch. The differences in S tell us that in this analysis Hipparchus is indicated with about a 7.8-sigma significance level. Schaefer pointed out that Rawlins' result depends critically on both his sample of selected stars and on his assumed value of *k*.

Next, we summarize Schaefer's analysis. He chose for the input sample of stars essentially all stars in the *Bright Star Catalog*[6], combining close neighbors that would be visually indistinguishable. Schaefer also assumed a minimum value for aerosol scattering based upon the best visibility conditions at sea level today in the areas around Rhodes and Alexandria. For three quadrants of the sky (right ascensions in the range $0° < \alpha < 270°$, and declinations less than -10°) he found $S = 667.4$ for Hipparchus and $S = 615.5$ for Ptolemy. For the fourth quadrant in right ascension he found $S = 176.2$ for Hipparchus and $S = 182.7$ for Ptolemy. The differences in S tell us that for the first three quadrants Ptolemy is indicated with about a 7-sigma significance level, while for the fourth quadrant Hipparchus is indicated with about a 2.5-sigma significance level. Schaefer also found that his results are very robust to a multitude of reasonable variations of his input assumptions, as long as aerosol scattering stays above a minimum level.

I have independently repeated the calculations of both Rawlins and Schaefer, and have confirmed that both sets of calculations are technically correct: if you use their input assumptions, you do get their result. Further, I have used the generally more complicated model of Schaefer, which also allows variation in the P_{det} function, to analyze Rawlins' selected subset of 30 stars, and I again get substantially the same result as Rawlins originally published.

So what should we conclude from these analyses? If either is to be believed, we must have confidence in the input assumptions. I would like to point out in particular the following three assumptions:

1. a star is included in the catalog based *exclusively* on the probability that a star of its apparent magnitude is *visible* at a specific latitude. This makes no allowance for the possibility that an observer might include stars reported to him from other,

perhaps more southerly, locations, or that the observer might work harder to include stars at lower altitudes. This also does not take into account that each star in the catalog was not only *seen*, but its position was also *measured*. Anyone who has ever tried it will know that the latter is much harder than the former.

2. when analyzing a fixed area in the sky, the model assumes that every star in that area was observed at the *same* latitude. In order to find a composite catalog the analyst must carefully search different areas of the sky to see if different latitudes are indicated. This is exactly what Schaefer did, to find his three quadrants for Ptolemy and one quadrant for Hipparchus solutions. But if the catalog is truly composite, as many catalogs are, with multiple observers at multiple latitudes, the model cannot reveal that fact.
3. the test does not use any *other* information we might have about a particular star that might shed light on who observed that star.

To illustrate the impact of these assumptions, let us consider the case of Canopus (α Car and BN892 in the ASC). In 130 BC Canopus culminated at about 1.3° at $\varphi = 36°$. Its visual magnitude was –0.72 (presumably the same as today) but its apparent magnitude in Rhodes was about 5. In Alexandria in 137 AD Canopus culminated at over 6° and its apparent magnitude was about 1.4. So the log-likelihood for Ptolemy is much smaller than for Hipparchus. Yet we know for certain than Hipparchus *did* in fact include rising and setting information for Canopus in his *Commentary to Aratus*[7], and Vogt[8] was able to use these data to deduce the coordinates that Hipparchus must have had for Canopus. Further, the data that Hipparchus reported imply that his coordinates for Canopus contained rather large errors of about 5°, and amazingly enough, we find those same large errors *repeated*[9] in the star coordinates for Canopus that appear in the ASC (see Table 1 and Figure 1). Thus we have a case where the maximum likelihood test tells us that Ptolemy is favored over Hipparchus as the observer of Canopus, while we have additional information that is *not* used by the test that tells us exactly the opposite.

In order to see whether this is a harmless special case or a more general problem, let's take a close look at how the difference in *S* values of about 52 actually arises in

Schaefer's analysis of the first three quadrants – the area in the sky that provides the strongest pro-Ptolemy result. When I repeat the analysis using my input star catalog (which differs in details from Schaefer's), approximately the same parameter assumptions, and my computer program, I find a difference in *S* values of about 54, so we know we are both in general agreement (and other more detailed comparisons confirm this completely).

Consider first those stars in the sky that *do not* appear in the ASC. For Hipparchus and Ptolemy, these stars contribute to *S* about 278 and 269, so the difference of 9 is a 3-sigma effect in favor of Ptolemy. Not negligible, but a small part of the overall difference of 54. Therefore, we see that *most* of the pro-Ptolemy signal is coming from stars that were actually in the catalog, not from stars that were omitted.

If we look at the differences in *S* values for the 284 stars in this part of the sky that are also *included* in the ASC, on a star-by-star basis, we get the histogram shown in Figure 2. We notice that this histogram is nearly symmetric about zero, except for a tail[10] of stars at positive *S*. Indeed, we notice that if we compute the sum of the *S* values for all stars except the 13 with the largest positive *S* values, i.e. those that favor Ptolemy most, then that sum is very nearly zero. This means that a very large part (46 out of 54) of the pro-Ptolemy signal in this test is in fact arising from 13 specific ASC stars. These stars are listed in Table 2.

Of the 13 stars, 5 of them, BN805, 992, 995, 996, and 997[11] *also* appear in Hipparchus' *Commentary to Aratus*, and like Canopus, each[12] has large common errors in both the *Commentary* and in the ASC (see Table 1 and Figure 1). We can therefore be pretty certain that these five stars, which are contributing a total of 20 to *S*, are in fact, like Canopus, giving us contradictory signals: a pro-Ptolemy signal from the visibility test, but a pro-Hipparchus signal from the coordinate errors (remember, the only information the visibility model takes from the ASC is whether or not a star is included – the actual coordinates and magnitudes listed in the ASC are not used in any way).

How deep does this problem reach? Without further independent analysis, we can only speculate, but the following line of thought is not unreasonable: let us consider whether the other eight stars in our signal might have been also copied. We know that BN805 (θ Eri) was copied, which at least suggests that BN803 and BN804, nearby neighbors in Eridanus, are also good candidates for copying. We know that 4 Ara stars, BN992, 995, 996, and 997 were copied, which suggests that BN994, also in Ara, might also be copied. That leaves BN884, 885, 887, 889, and 893, all in Argo Navis. Now we know that Ptolemy copied at least two stars from Argo Navis: 892 (Canopus) and 918 (π Hya), but these stars did not make our list of 13 'critical' stars. Still, it might be taken to suggest that Ptolemy copied others from Argo, further weakening the case against Hipparchus. In fact, a simple model analysis[13] of the size of the correlations between the Commentary and Almagest errors suggests that a large fraction, even up to 100%, of stars common to the Commentary and the Almagest were copied, so these speculations are far from groundless. All in all, then, we have either direct or circumstantial evidence that a very large part of the pro-Ptolemy signal issued by the visibility test is, in fact, contradicted by the coordinate error data.

How should we resolve this dilemma? One way out was recently offered by Schaefer[14], who points out that we need merely assume that Ptolemy did everything he claims, i.e. look at the sky and measure the positions of the stars, but then perhaps compares his results with old records he had from Hipparchus and for some reason included Hipparchus' coordinates for some subset of the stars instead of his own measurements in the ASC. This scenario thus uses in a crucial way the model assumption that the only issue being tested, and hence the only conclusion that can follow, is whether a given star was *observed* at a particular latitude. It would be interesting to try and further test this scenario, but I don't presently know how to do that.

Another option is to incorporate into the maximum likelihood calculation the *a priori* knowledge that *some* stars were definitely observed and measured by Hipparchus and copied by Ptolemy. For those stars it makes little sense to blindly apply the basic model assumption that every star is included in the catalog with probability P_{det}. Indeed, for those stars the statistically sound procedure would be to say that P_{det} is simply unity for Hipparchus and zero for Ptolemy (or perhaps use a gaussian probability distribution sharply peaked at the parameters implied by Hipparchus as the observer). In that case, however, the likelihood $L = \prod_{i=1}^{N} P_i$ will obviously be sharply maximized for Hipparchus, *no matter what* the contributions of the other stars (unless someone can find a star that is *known* to be measured by Ptolemy and not by Hipparchus – so far, not a single such star is known). The reader might complain, correctly, that this makes the whole question default to Hipparchus, but the real reason this happens is the model assumption that *all* the stars with a fixed region of the sky were measured at the *same* latitude. So in fact, the default is built into the model.

It appears to me that we must ask which conclusion do we trust the most, which in turn means which set of underlying assumptions is most likely to be true in *this specific case*. I know of no reason to mistrust the evidence from the large shared errors, but we must admit that only five of the crucial 13 stars are virtually certain to be of Hipparchan origin. The evidence that the remaining eight were also copied is, strictly speaking, circumstantial and statistical. On the other hand, the discussion above makes it clear that the fundamental assumptions that underlie the visibility test may not be nearly so solid, at least in the case at hand. Certainly the simplest resolution is that the visibility test, as implemented, just doesn't work for the ASC. It would be interesting if someone could find an objective way to distinguish these options.

ACKNOWLEDGEMENT

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Name	BSC	Bailey Number	Type	Commentary Error	Almagest Error
θ Gem	2540	426	1	4.06	4.04
	2540	426	2	3.03	3.24
ι Can	3474	455	1	-5.72	-3.04
	3474	455	2	-3.17	-3.61
β Sgr	7337	592	3	-7.34	-5.74
	7337	592	4	-4.92	-3.94
θ Eri	897	805	1	-2.54	-2.61
	897	805	2	-2.91	-3.42
	897	805	3	5.75	7.06
	897	805	4	6.76	8.28
α Car	2326	892	3	5.11	4.69
	2326	892	4	5.03	5.25
π Hya	5287	918	1	3.48	3.07
	5287	918	2	3.65	3.45
	5287	918	3	-6.52	-7.52
	5287	918	4	-3.75	-4.39
α Cen	5459	969	1	4.73	4.74
	5459	969	2	6.79	6.33
θ Ara	6743	992	1	-1.62	-2.96
	6743	992	2	-2.53	-3.53
γ Ara	6462	995	3	-7.89	-8.80
	6462	995	4	-5.91	-5.84
β Ara	6461	996	3	-12.72	-8.69
	6461	996	4	-9.01	-5.55
ζ Ara	6285	997	1	-1.30	-1.15
	6285	997	2	-1.05	-1.37

Table 1. The stars common to both the Commentary and the Almagest that either have large shared errors or which play a role in the visibility test.

Name	Bailey number	S_{Hipp}	S_{Ptol}	S_{Hipp}-S_{Ptol}
1195	803	3.47	1.76	1.71
1143	804	6.94	4.28	2.66
θ Eri*	805	5.44	0.19	5.26
χ Car	884	4.63	1.07	3.57
ο Vel	885	2.75	0.96	1.79
V344 Car	887	14.80	8.09	6.71
N Vel	889	3.23	0.54	2.69
τ Pup	893	3.60	0.27	3.34
θ Ara*	992	2.73	1.07	1.65
ε Ara	994	4.88	2.82	2.06
γ Ara*	995	9.74	2.34	7.40
β Ara*	996	4.09	0.43	3.66
ζ Ara*	997	4.06	0.74	3.32

Table 2. A number in column 1 gives the star's ID in the *Bright Star Catalog*. The number in column 2 gives the star's ID in the *Almagest* star catalog. S_{Hipp} and S_{Ptol} are the contributions of that star to the log-likelihood assuming Hipparchus and Ptolemy as the observer, respectively. The stars marked with * have large shared errors in both the *Commentary* and the ASC, and hence we can be fairly certain that Ptolemy copied them from Hipparchus.

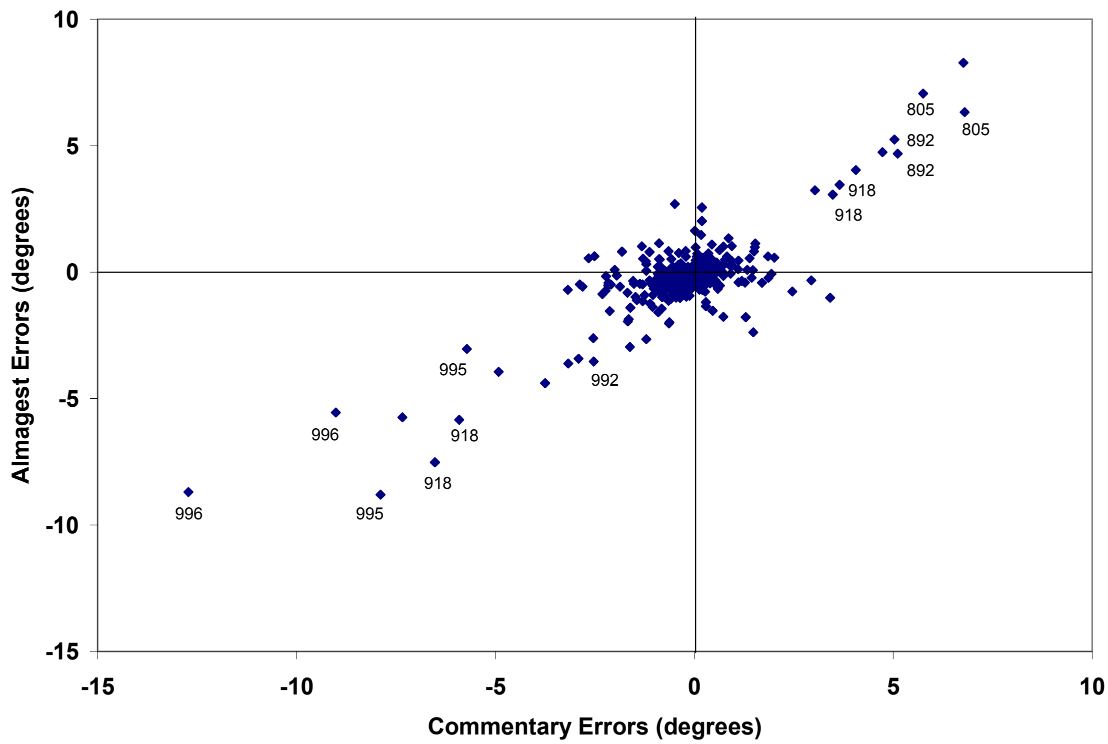

Figure 1. A scatter plot showing the correlation of the Commentary and Almagest errors for phenomena of types 1-4. Those stars with large shared errors that are discussed in the text are marked with their Bailey number (column 3 in Table 1).

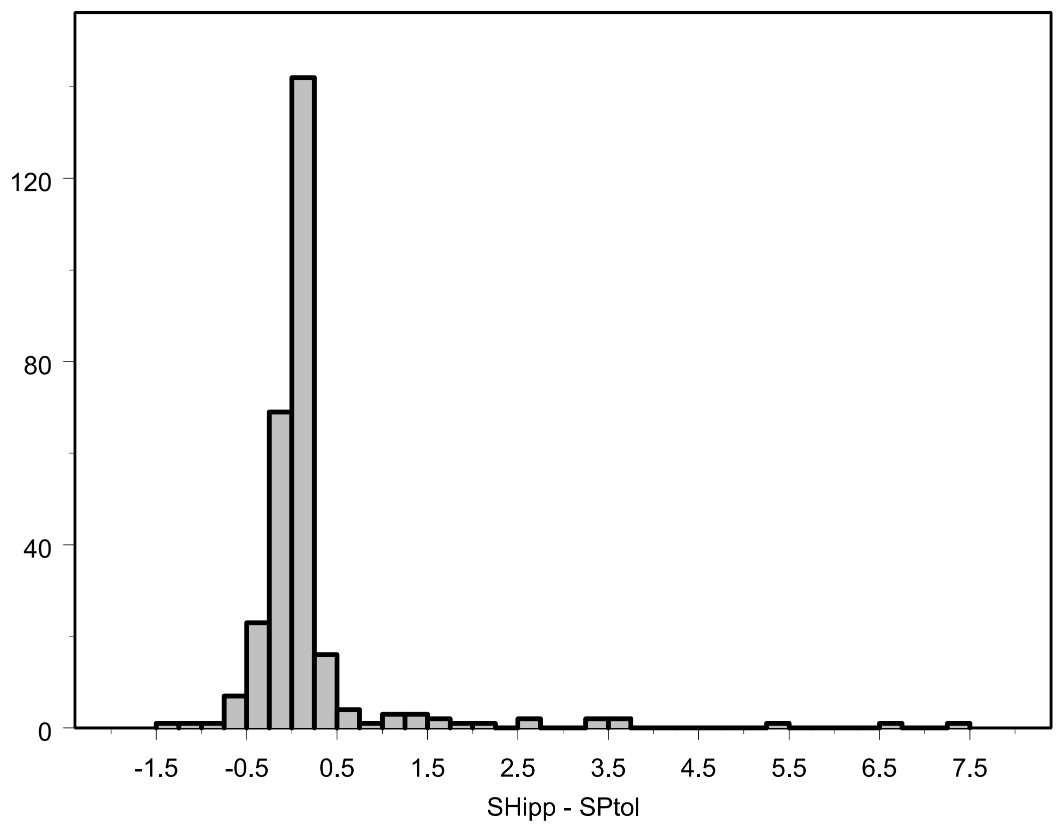

Figure 2. The distribution of *S* differences for the stars that are in the ASC.

FOOTNOTES

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[1] *Ptolemy's Almagest*, transl. by G. J. Toomer (London, 1984).

[2] J. B. B. Delambre, *Histoire del'astronomie ancienne* (2 vols, Paris, 1817), ii. 261-4.

[3] D. Rawlins, "An investigation of the ancient star catalog", *Publications of the Astronomical Society of the Pacific*, xciv (1982), 359-73.

[4] B. Schaefer, "The latitude of the observer of the Almagest star catalogue, *Journal for the history of astronomy*, xxxii (2001), 1-42.

[5] B. Schaefer, "Astronomy and the limits of vision", *Vistas in Astronomy*, xxxvi (1993), 311-61.

[6] D. Hoffleit, The bright star catalog (New Haven, 1997).

[7] Hipparchus, *In Arati et Eudoxi phaenomena commentariorium*, ed. and transl. by K. Manitius (Leipzig, 1894).

[8] H. Vogt, "Versuch einer Wiederstellung von Hipparchs Fixsternverzeichnis", *Astronomische Nachtrichten*, ccxxiv (1925), cols 2-54.

[9] G. Grasshoff, *The history of Ptolemy's star catalogue* (New York, 1990).

[10] The reader might wonder whether this tail is peculiar to the case at hand, or a general feature that should be expected. Monte Carlo simulation confirms the second possibility. Indeed, I have generated hundred's of synthetic star catalogs by extracting with probability P_{det} stars from the Bright Star Catalog. When these synthetic catalogs are analyzed, distributions very similar to that shown in Figure 1 always result. Indeed, it is fairly obvious that when the model indicates a southern observer, the reason will always be that the northern observer was penalized for including too many dim, low altitude stars. Conversely, when a northern observer is indicated, it will be because the southern observe *omitted* too many bright stars.

[11] Manitius and Grasshoff identified the first star to rise in Ara as ε Ara (BN994), but the surrounding textual and astronomical evidence in the *Commentary* establishes beyond any reasonable doubt that the correct identification is ζ Ara (BN997).

[12] G. Grasshoff, *op. cit.* (Ref. 9), 331-34.

[13] D. Duke, "Associations between the ancient star catalogs", *Archive for the History of the Exact Sciences*, (forthcoming).

[14] B. Schaefer, “The Great Ptolemy-Hipparchus Dispute”, *Sky & Telescope*, 103 (February 2002), 38-44.