Correcting Asymmetric Lens Distortion with Thin Plate Splines

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Abstract

Many areas of research and industry necessitate the collection of landmark points from photographs. Due to design and manufacturing imperfections in camera lens elements, coordinates of points in photographs appear displaced from the desired projection. Common methods of correcting lens distortion assume that displacements are symmetric around the center of the photograph, and may not capture the full nature of the distortion.

Previously, the author has used thin plate splines to attempt to solve the problem of asymmetric lens distortion. The goal of this project is to enhance and improve that work by developing cross-platform, user-friendly, and automated software to correct lens distortion. Foremost among the improvements include an automated point detection scheme, which applies difference of gaussians convolution, balanced histogram thresholding, and k-means clustering to detect calibration landmarks. This removes the need for the tedious and error-prone process of selecting calibration points by hand, thus increasing the practical number that may be used. The computationally expensive image unwarping process has also been parallelized and tuned, achieving a far more reasonable execution time. Here, the improvements are explained and demonstrated, and an analysis of the methods accuracy with a greater number of control points is made.

Introduction

Imperfections in the design and manufacture of camera lens elements cause points in a photograph to appear displaced from their expected rectilinear projection due to a phenomenon called lens distortion. Common methods of correcting lens distortion in photographs, including the first method introduced by Brown in his 1966 paper *Decentering distortion of lenses*, make the assumption that distortion is symmetric about the center of the photograph.

By photographing a configuration of points with known geometry, it is possible to approximate the distortion function with a thin plate spline radial basis function mapping. The local nature of the thin plate spline is well suited to approximating asymmetric lens distortion.

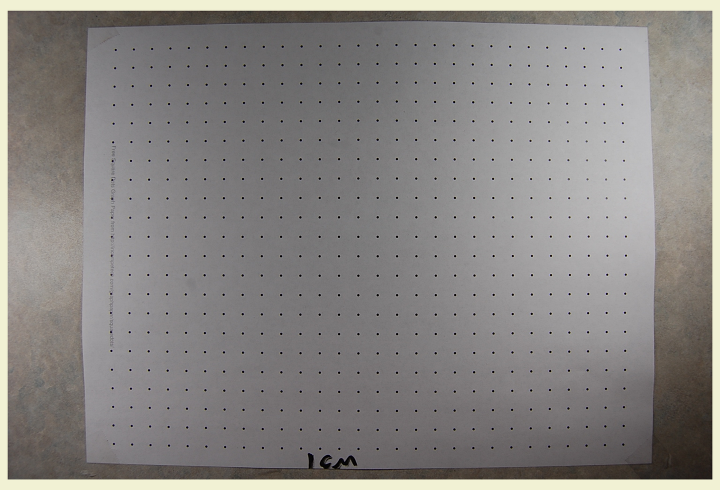
Automated Calibration Point Detection

In order to perform the thin plate spline warping, the coordinates of control points in the calibration image must first be known. In the early stages of this project, control points were located manually with a mouse. The author desired a less tedious and error-prone process, and devised an automated point detection scheme.

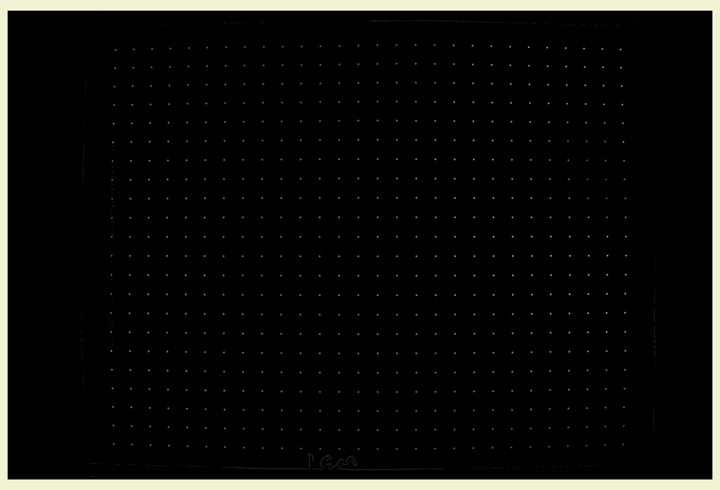
1.)The original image (Fig 1a) is convolved with a Difference of Gaussians filter. This acts as a spatial band-pass filter by subtracting a somewhat blurred version of the image from an even more blurred version of the image (Chen et al., 1987), removing unwanted noise and leaving only the control points (Fig 1b).

2.) The image is segmented into control points and background with balanced histogram thresholding (Fig 1c). This technique, first developed to segment images of DNA microarrays, efficiently segments images with bimodal histograms (Anjos and Shahbazkia 2008).

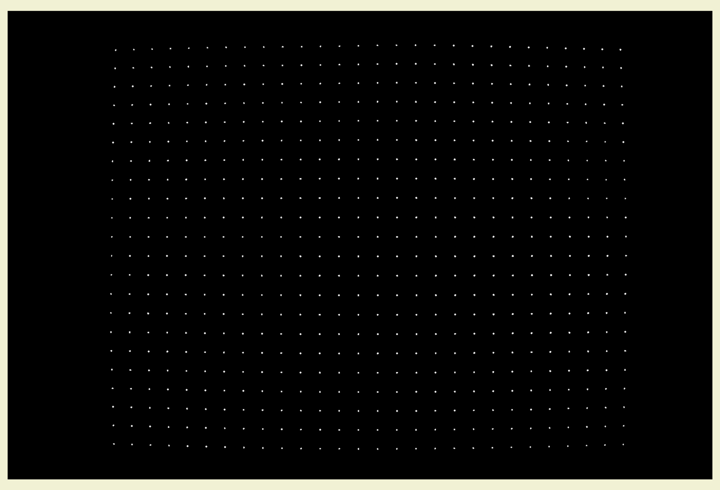
3.) The user is prompted for the location of the top-leftmost control point, as well as the number of rows and columns of control points. This information is used to generate a grid of the approximate locations of the control points. These points are then used as initial seeds in a k-means clustering algorithm which yields the coordinates of the control points on the image (MacQueen 1967) (Fig 1d).



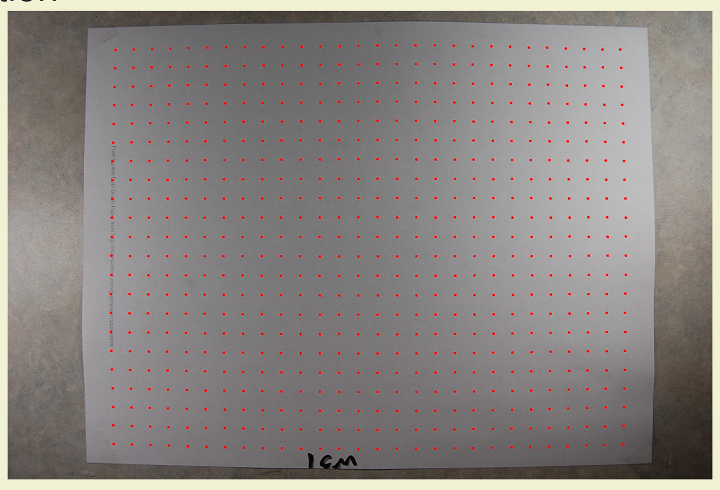
(a) Original Calibration Image



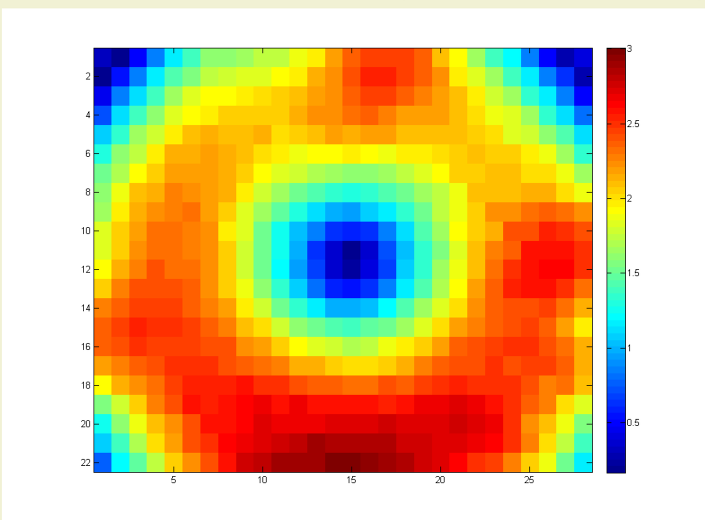
(b) After Difference of Gaussians Convolution



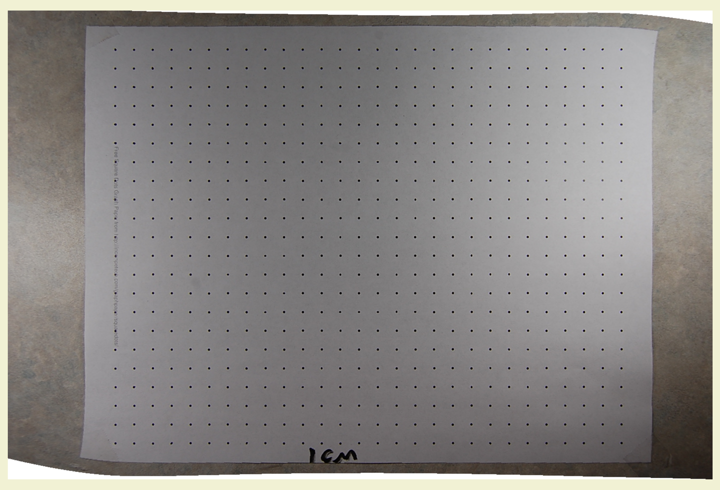
(c) After Balanced Histogram Thresholding



(d) After K-Means Clustering



(e) Heatmap of Distortion at Control Points (in mm)



(f) After TPS Correction

Fig 1.

Thin Plate Spline Distortion Correction

Imagined as the deflection of points on a thin sheet of metal, thin plate splines are used to apply a spatial mapping from one set of points exactly to another, while interpolating the values in between (Bookstein 1989). By photographing an object with known geometry (here, a piece of paper with regularly-spaced dots) and comparing the resulting photograph with the "ideal" representation, thin plate splines can be used to correct the distortion present in the photograph. Given that the camera configuration remains the same, subsequent photographs can be corrected using the same parameters. The so-called "bending energy" is minimized so that only one solution exists.

The x and y displacements are computed separately:

$$f(x, y) = a_1 + a_x x + a_y y + \sum_{i=1}^{n} w_i U(|P_i - (x, y)|)$$

The unknown parameters are determined by solving the linear system

$$\begin{bmatrix} K & P \\ P^T & O \end{bmatrix} \begin{bmatrix} \vec{w} \\ \vec{a} \end{bmatrix} = \begin{bmatrix} \vec{v} \\ \vec{o} \end{bmatrix}, \text{where } K = \begin{bmatrix} 0 & U(r_{21}) & \cdots & U(r_{1n}) \\ U(r_{21}) & 0 & \cdots & U(r_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ U(r_{n1}) & U(r_{n2}) & \cdots & 0 \end{bmatrix}, P = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & y_n \end{bmatrix}, r_{ij} = |P_i - P_j|,$$

An "ideal" grid, with the same number of control points, is superimposed onto the distorted grid by centering it on the mean of the distorted points and rotating it to minimize the sum of squared distances between the two configurations. This process is known as as Ordinary Procrustes Analysis (with scale re-introduced) (Boas 1905) and determines where the distorted points will map to.

Due to the nature of thin plate splines, the condition number of the L matrix grows with a large number of points, and also when pairs of data sites occur close together (Sibson and Stone 1991). In order to prevent the condition number of L from becoming unmanageably large (in some cases over 10E22), both the ideal and distorted grids are scaled to the unit interval before warping, then back afterward.

Previously, this computation was extremely slow, taking nearly 20 minutes to warp a 1504x1000 pixel image with 200 control points. Now, with optimized matrix algebra routines and a parallel thread pool to compute the TPS offsets, it only takes 30 seconds to warp an image with 616 points on a dual core, 2.1 GHz workstation.

Because thin plate splines map the control points exactly from the distorted grid to the ideal grid, it is possible to measure the effect of distortion at those points, but illogical to measure post-correction error there. For this reason, post-correction error was evaluated by using only half of the available points to warp the image, then measuring the error at the points in between. The camera was mounted on a Camstand(tm) copy stand, and placed as close to the test page as possible (approximately 1.5 feet). The stand rested on a level table and the camera was leveled before each photograph was taken. The test image was generated from an online graph paper generator website and consisted of a grid of 22 by 28 points, each 1 cm apart (MacLeod, 2013).

Results and Discussion

The test image revealed a mean error of 7.92 pixels at the control points, as well as a minimum error of 0.66 pixels and maximum error of 12.03 pixels, or 1.98, 0.165, and 3.01 millimeters respectively. To test the effectiveness of the distortion correction method, half (308) of the total points, sampled in a checkerboard pattern, were used to unwarp the image, and the error after correction was measured at the other half of the points. This yielded a mean error of 0.58 pixels, a minimum error of 0 pixels, and a maximum error of 6.32 pixels, or 0.145, 0, and 1.58 millimeters respectively. All but 3 of the measured points had error below 2 pixels or 0.05 mm, and nearly half (148) had error less than or equal to 1 pixel or 0.025 mm.

Despite the relatively high computation time compared to other methods, the thin plate spline method of lens distortion correction achieves merit in its high accuracy and ease of use. The heat map of the distortion (Fig 1e) shows clearly that lens distortion is not necessarily a radially-symmetric phenomenon, and thus methods that do not account for this asymmetry may be inadequate. This distortion correction method may prevent unexpected bias in metric data collected from photographs, such as when digitizing landmarks on specimens for geometric morphometric research.

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