Exploring a Multi-Resolution Approach within the Shallow-Water System

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Abstract The goal of this project is to explore how multi-resolution grids effect global shallow-water models. Our approach is to generate high-quality Spherical Centroidal Voronoi Tessellations (SCVTs) that result in two dominate resolutions, where one resolution (the fine scale) covers the region of interest and the other resolution (the coarse scale) covers the rest of the sphere. The two dominate scales are separated by a mesh transition zone that, like the fine and coarse scales, is easily altered through the user-specified mesh-density function. We apply our finite-volume solver along with these variable resolution SCVTs to the standard suite of shallow-water test cases in order to highlight the strengths and weakens of this multi-resolution approach.

Goal of Research

We are attemping to create variable resolution grids, that preform comparably to uniform resolution grids. These new girds could allow us to obtain a 'better' solution in a region of the grid, and are well suited for regional modeling. One thing to note, is that we are not looking for grids that produce a better solution, as we don't expect this to happen, we are simply looking for grids that don't preform significantly worse than an uniform grid. Later we could attemp to tailor the grids to specific parameterizations of other physical processes, to obtain a better result than the parameterized version of the uniform grid, especailly in a specific region. We will be using spherical centroidal Voronoi tessellations for our grids.

Mathematical Model

This research is using a newly developed finite volume solver, and the shallow water equations for the testing purposes of our grids. The shallow water equations are given as follows:
Momentum Equations:

$$u_t + uu_x + vu_y - fv = -gh_x$$
$$v_t + uv_x + vv_y + fu = -gh_y$$

Continuity Equation:

$$h_t + (hu)_x + (hv)_y = 0$$

Density function for generators

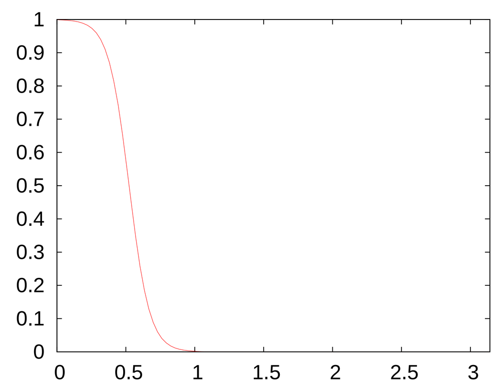
To create our variable resolution grids, we had to come up with a density function that would be used to generate our SCVT. The criteria we were looking for was:

- Radially Symmetric
- Smooth Transition Region
- Maximum Value of 1

The density function that we came up with is as follows:

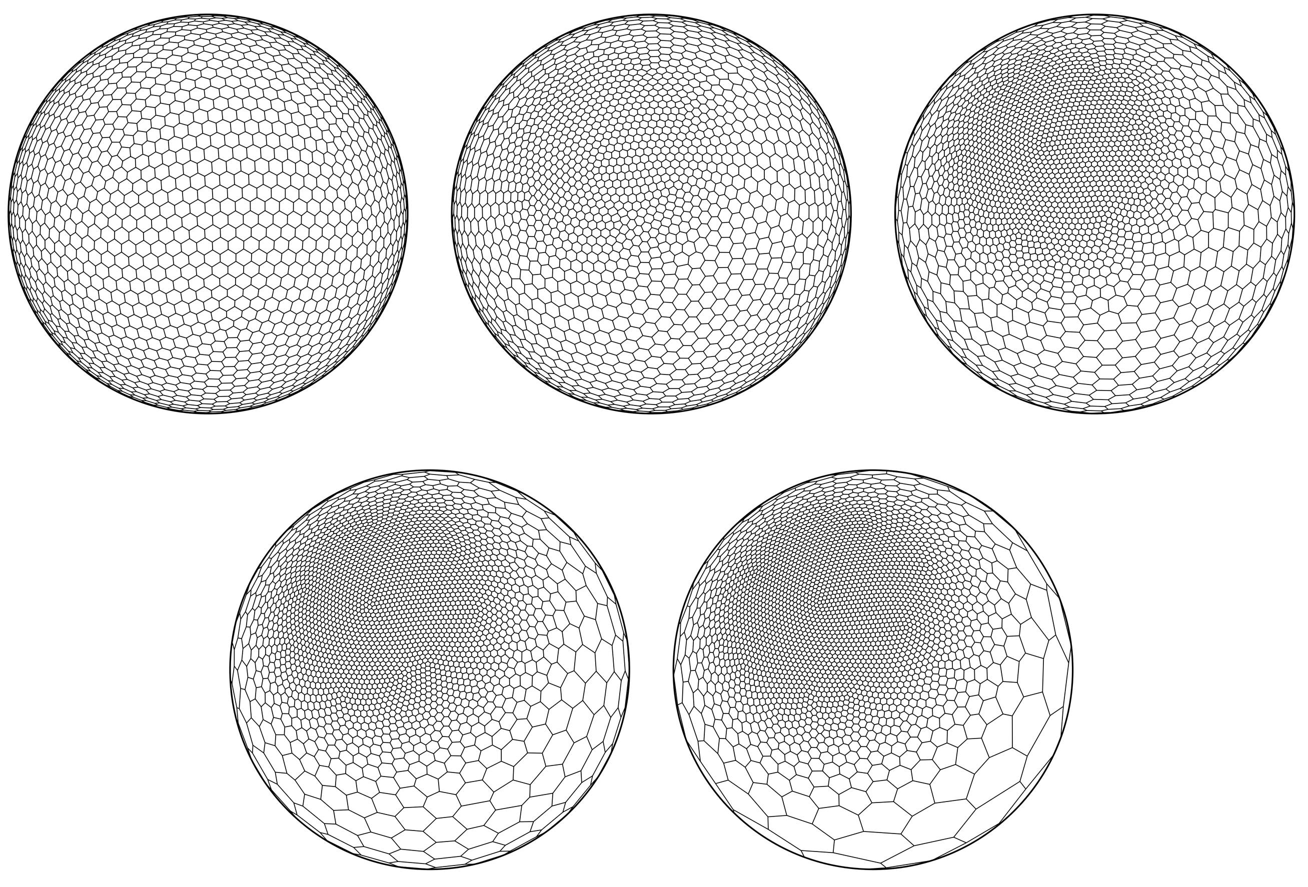
$$\rho = (((\tanh(t_c - r) * (1.0/w)) + 1.0)/2.0) * (1.0 - m_v) + m_v$$

In this equation t_c is the center of the transition region (in radians) from the center of the density function, r is the distance in radians from the center of the density function, w is a paramter to control the width of the transition region, and m_v is the minimum value of density this function should output. Using the paramters of $t_c = \frac{\pi}{6}, w = 0.15, m_v = 0.0001$ gives us a denstiy function that can be seen below.



Grids for Simulation

For our test cases, we generated SCVTs with four different numbers of generators. The numbers of generators used were 2562, 10242, 40962, and 163842. These correspond to grid spacings of about 480km, 240km, 120km, 60km respectively. For the varaible resolution grids, we wanted a range of grids we could test. By changing m_v we could control the grid spacing difference between the coarse region and the fine region by some factor. We generated 5 different grids at each of the grid point, using m_v to give us a factor of 1, 2, 4, 8, and 16 between the coarse and the fine regions. When the factor is 16 it means that the fine region cells are about 16 times smaller than the cells in the coarse region. The different factors produce grids that look like the following:



Shallow Water Test Cases

With each of the grids, we preform simulations using some of the standard shallow water test cases as defined in [1]. To compare, we are using Test cases 2, 5 and 6.

- **Test Case 2**: Global steady state nonlinear geostrophic flow
- **Test Case 5**: Zonal flow over an isolated mountain
- **Test Case 6**: Rossby-Haurwitz wave

Because test case 2 is steady state, we are comparing the simulation after 12 days to it's initial conditions. Test cases 5 and 6 are compared to high resolution spectral element method simulation results for each of their test cases and are run for 15 days. The results will also be compared with [2]

Preliminary Results

The error norms used are defined as:

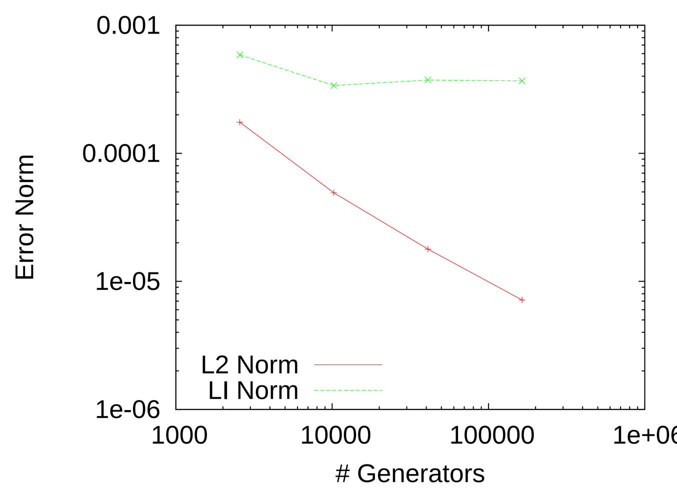
$$L_2 = \frac{(S[(f_n(j) - f_r(j))^2])^{\frac{1}{2}}}{(S[(f_r(j))^2])^{\frac{1}{2}}}$$
$$L_I = \frac{\max_j |f_n(j) - f_r(j)|}{\max_j |f_r(j)|}$$

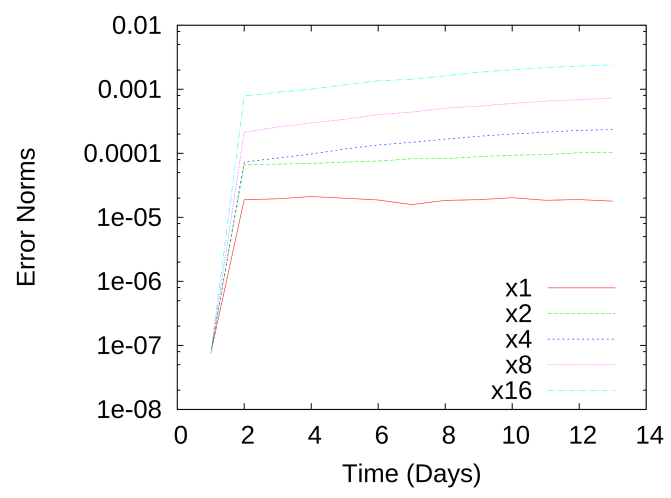
Which is used so we could directly compare with [2]. In this, $S[]$ is defined as:

$$S[f(j)] = \frac{\sum^(N_j)_(j=1) f(j)A(j)}{\sum^(N_j)_(j=1) A(j)}$$

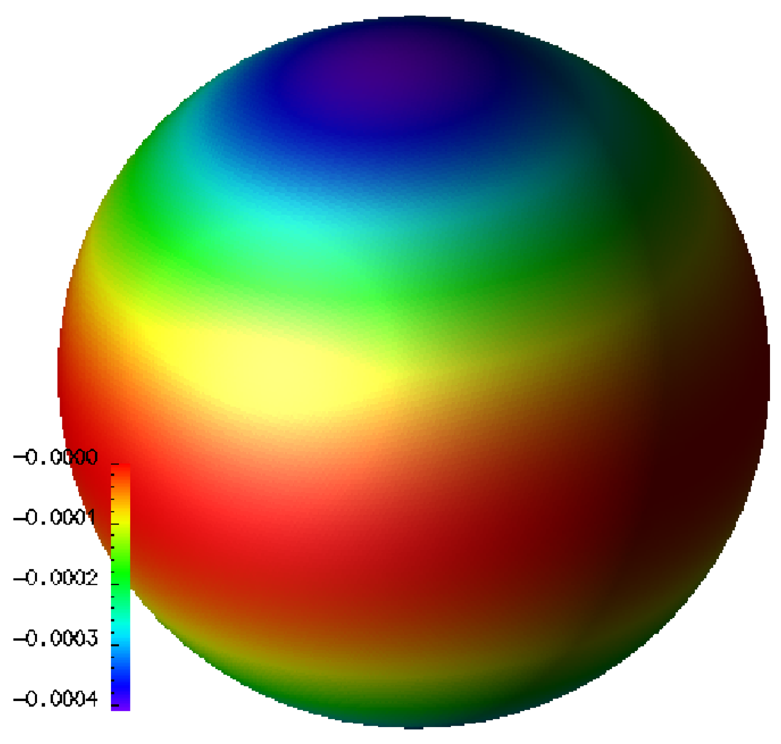
In these, f_n is the numerical solution, f_r is the reference solution, and $A(j)$ is the area of the current cell.

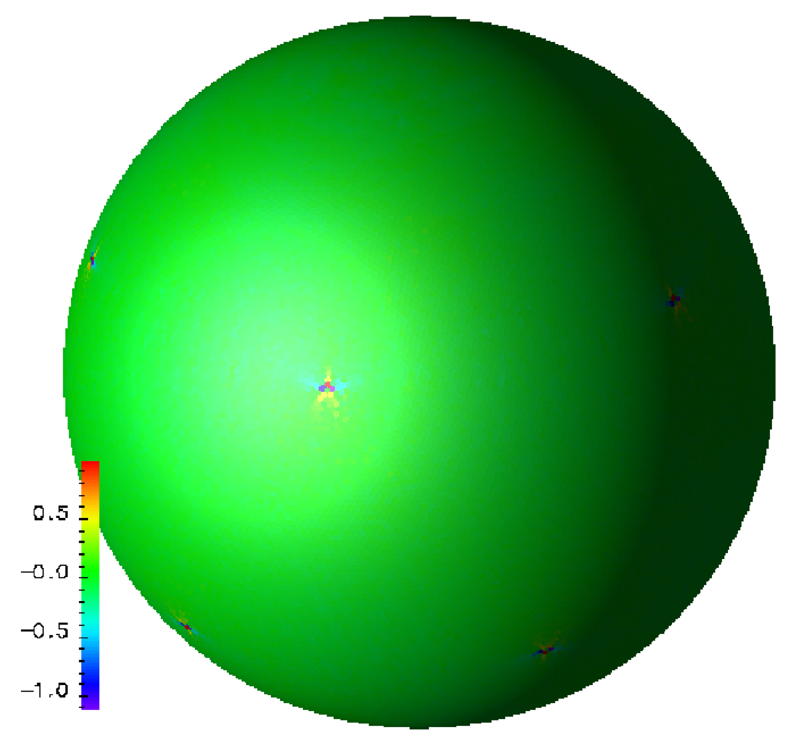
The results presented here are relative to test case 2. We started with a convergence study, and after the convergence was shown to be comparable to [2], we computed the errors as a function of time in days for test case 2 on all 5 of the variable resolution grids. Both of these can be seen below.





Of course, the x16 grid preforms worse than the uniform grid, but it doesn't appear to be significantly worse, and possibly may bet better than the uniform mesh after some parameterization is applied to the grids. The Distribution of error on the uniform grid can be seen below, at the initial condition and at the end of the simulation.





Future Work

- Further Analysis of Variable Resolution Grids
- Explore other Test Cases
- Grid based parameterization studies of physics
- Possible finite element solver implementation

References

[1] D. Williamson, J. B. Drake, J. J. Hack, R. Jakob, P. N. Swartzrauber, Journal of Computational Physics **211**, 102, (1992).

[2] T.D. Ringler, J. Thuburn, J. B. Klemp, W. C. Skamarock, Journal of Computational Physics **3065-3090**, 229, (2010)