

# Multi-Compartmental Modeling of Astrocytes

Gordon Erlebacher  
Wed. Nov. 18, 2015

# Research Group

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- Joel Tabak, (Univ. Exeter, UK)
- Nathan Crock (Ph.D.)
- James Schummers (Max Planck, FL)
- Monica Hildago (Max Planck, FL / Mexico)
- Gordon Erlebacher

# Roadmap

- Motivation
- Biology (Neurons, Synapses, Astrocytes)
- Single point astrocyte model
- Multi-compartmental astrocyte model
- Results
- Future extensions



## James Schummers, PhD

Research Group Leader

**Cellular Organization of Cortical Circuit Function**

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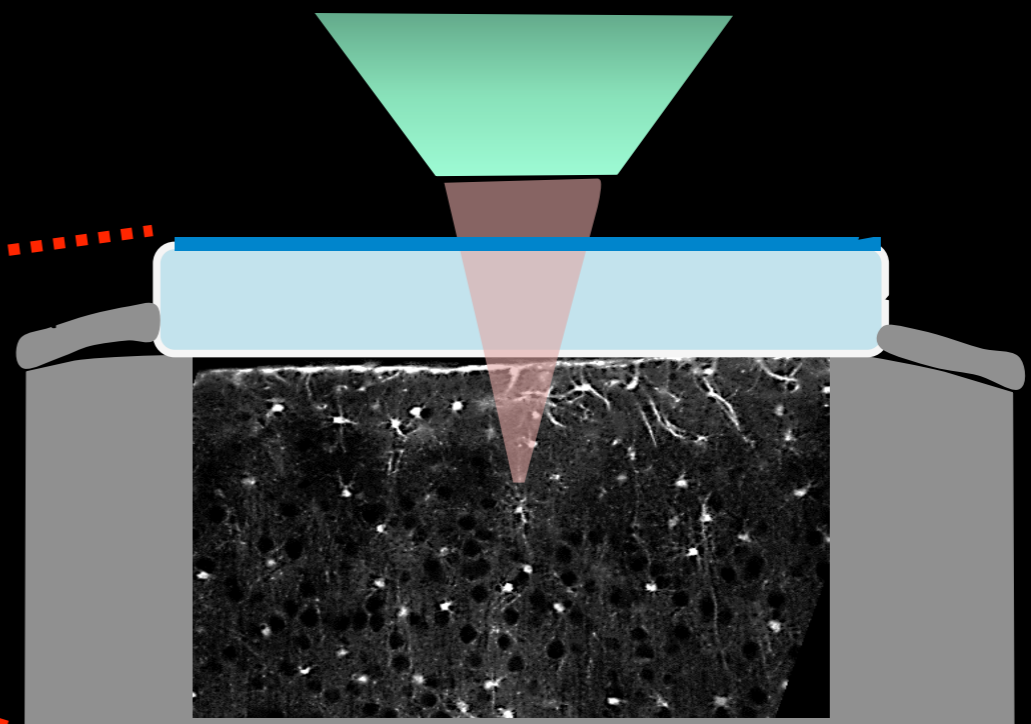
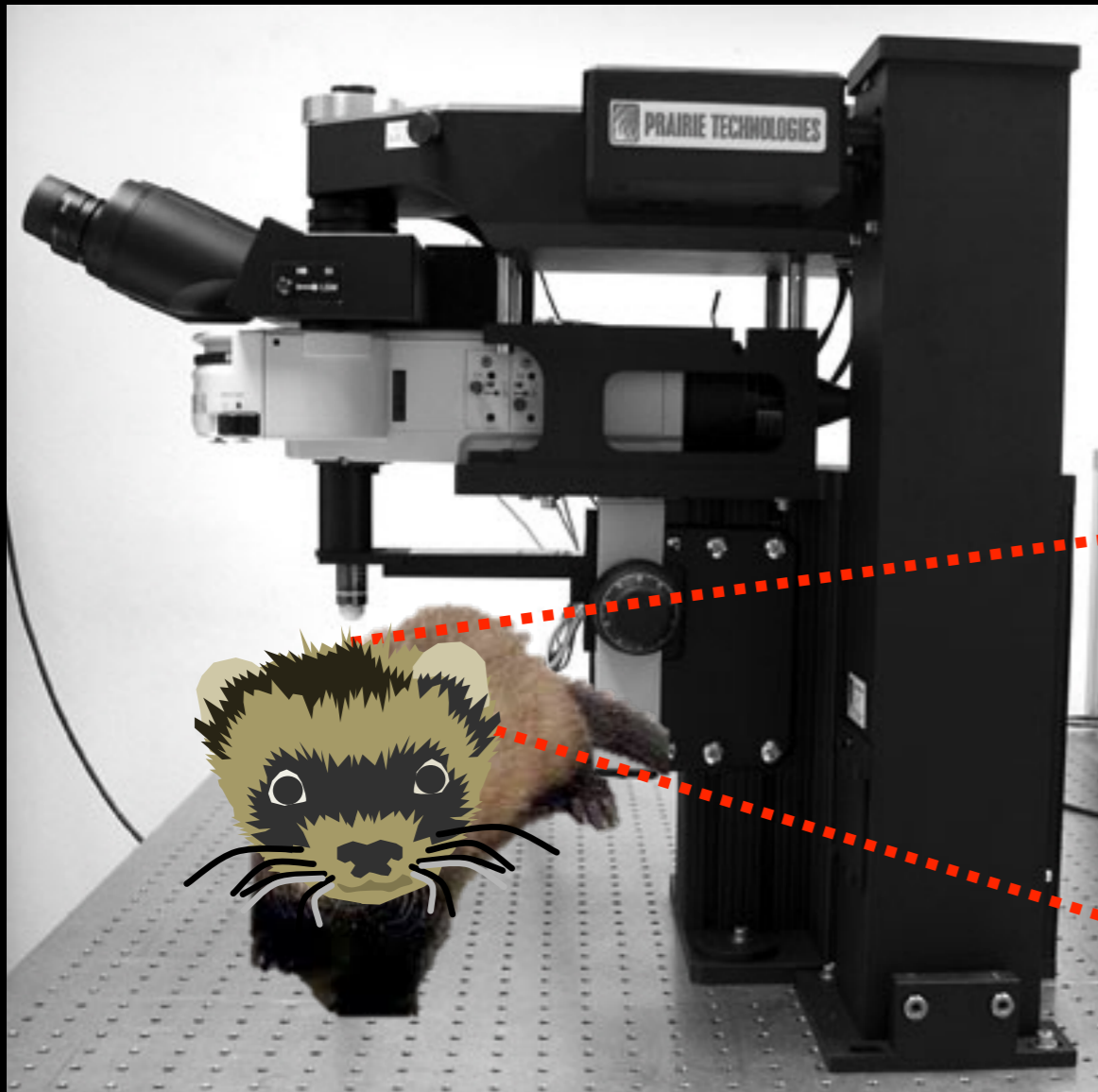
[Home](#) / [Our Science](#) / [Our Scientists](#) / James Schummers

## Researcher Bio

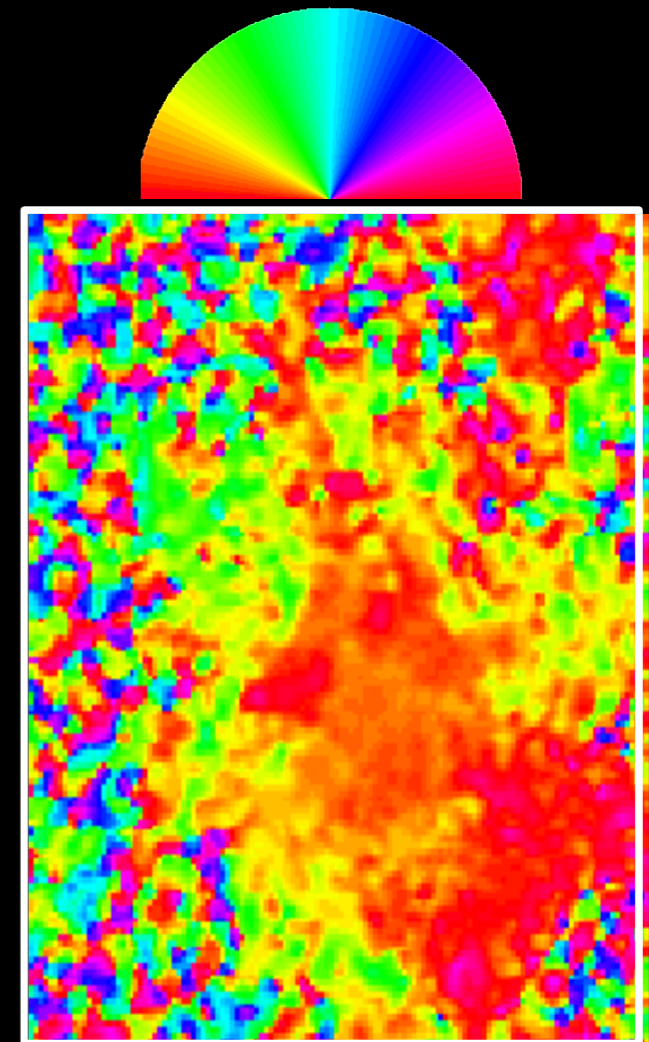
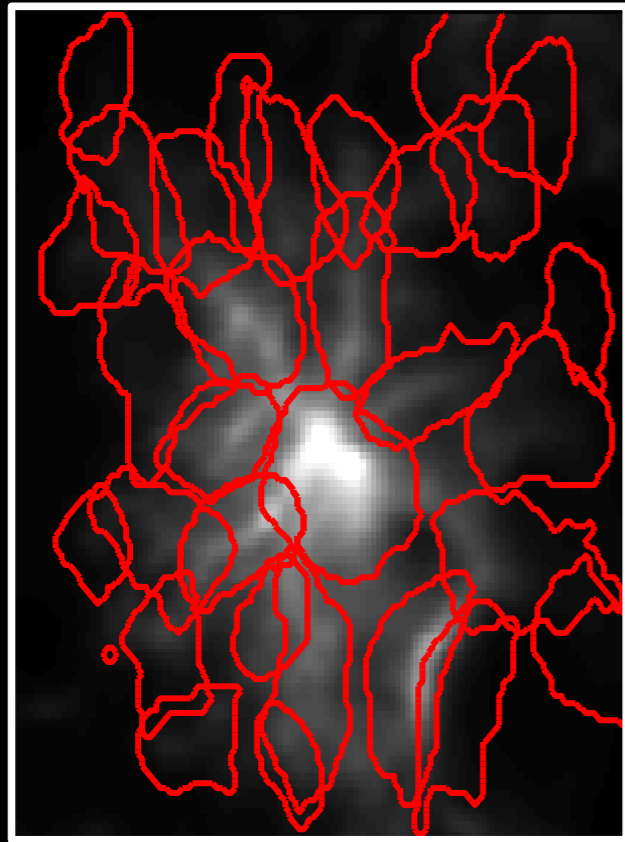
Dr. James Schummers was named an independent Research Group Leader at the Max Planck Florida Institute for Neuroscience in June 2010 and heads the Cellular Organization of Cortical Circuit Function research group. Dr. Schummers received his bachelor's degree in Neuroscience from Oberlin College in Oberlin, OH, where he studied the effects of the neurotransmitter neuropeptide-Y on long-term potentiation (LTP) in the hippocampus. He then moved to Denver CO, where he studied the effects of alcohol on LTP in the Department of Pharmacology at the University of Colorado Health Science Center. He received a PhD in Systems Neuroscience at the Massachusetts Institute of Technology with the support of a Howard Hughes Pre-Doctoral Fellowship. His thesis work combined intracellular and extracellular single neuron recordings with optical imaging approaches to study the integration of synaptic inputs in the context of visual processing. His postdoctoral work, also at MIT, focused on single-cell resolution imaging to study the response properties of different classes of cells, including both neurons and astrocytes, in the visual cortex.

# In vivo 2-photon imaging in ferret visual cortex

- in vivo two photon imaging
- Lightly anesthetized (isoflurane)
- Adult ferrets



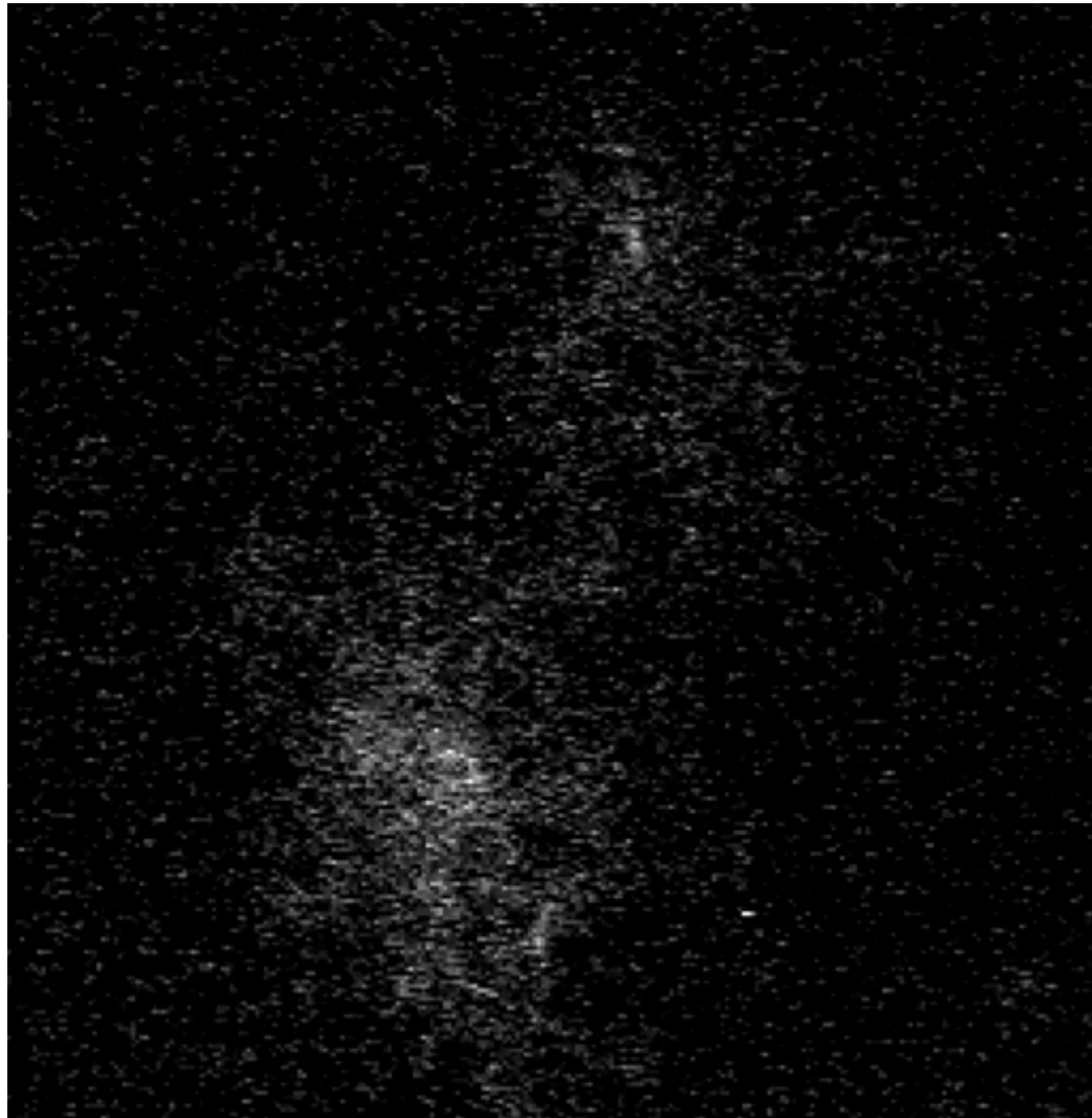
# Orientation tuning in subcellular domains

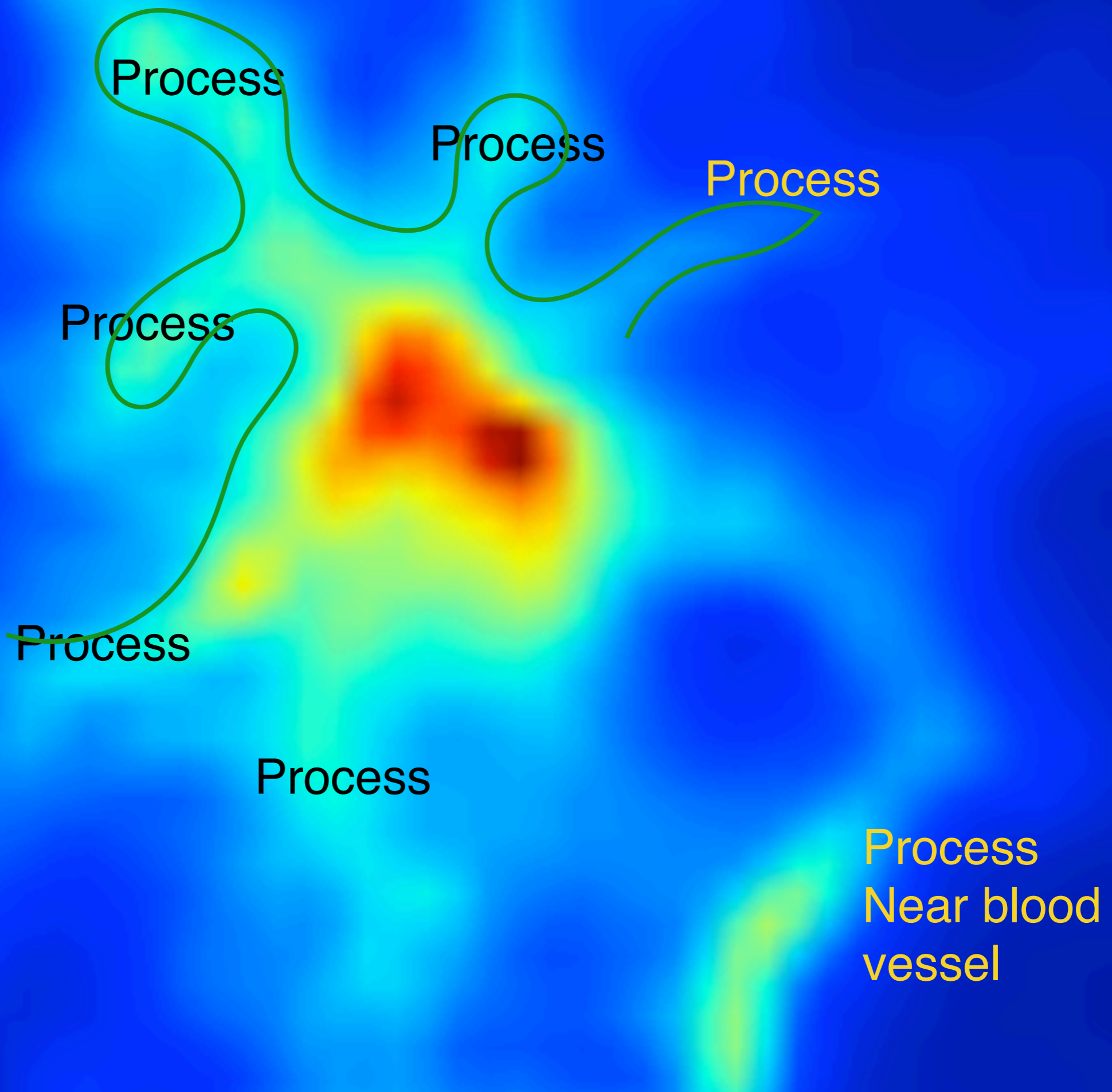


Each sub-domain has similar orientation tuning, with quantitative differences

Does this suggest that they are responding to distinct neural activity?

# 4Hz Animation: Raw Data, 256x256



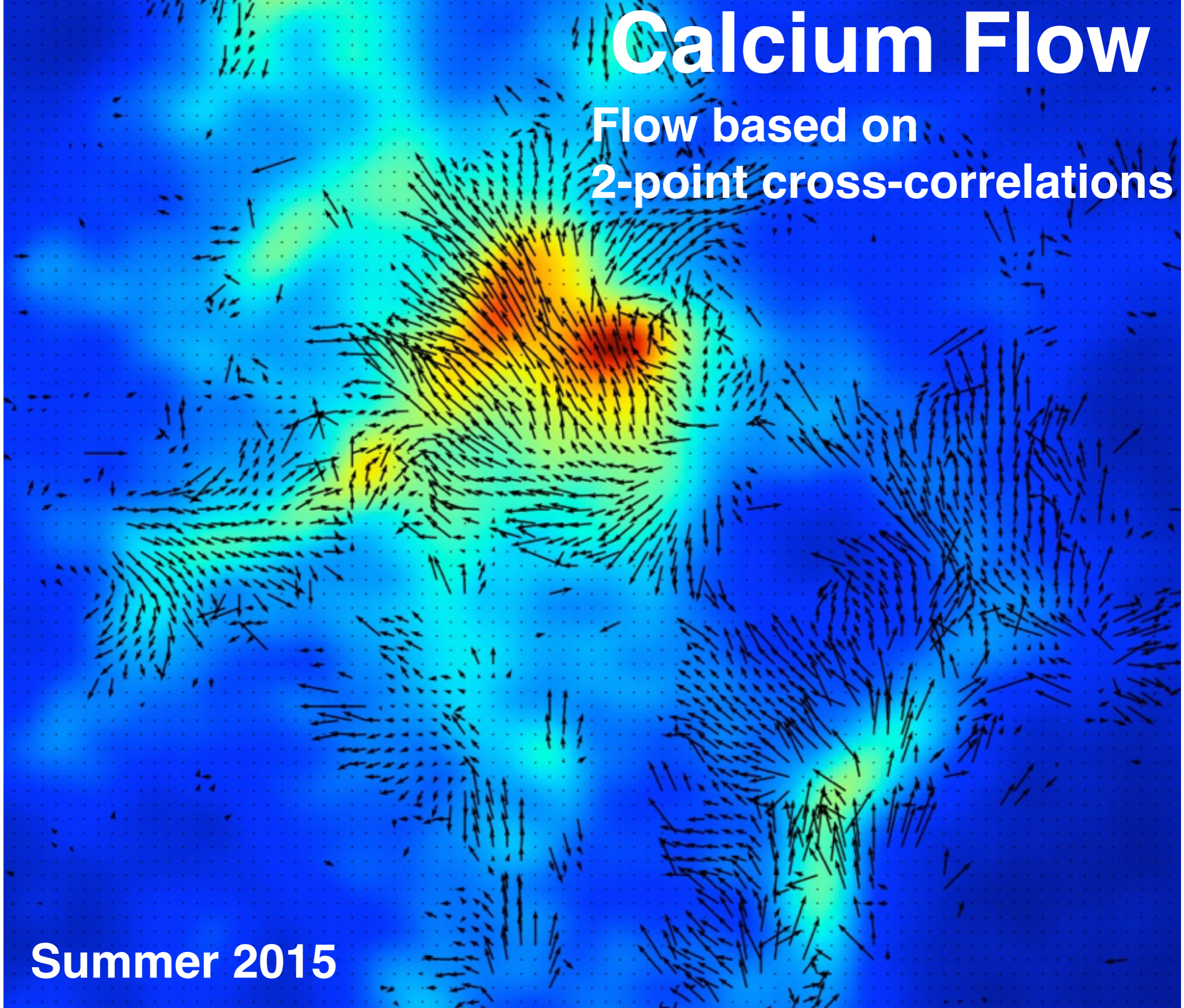




# Calcium Flow

Flow based on  
2-point cross-correlations

Summer 2015

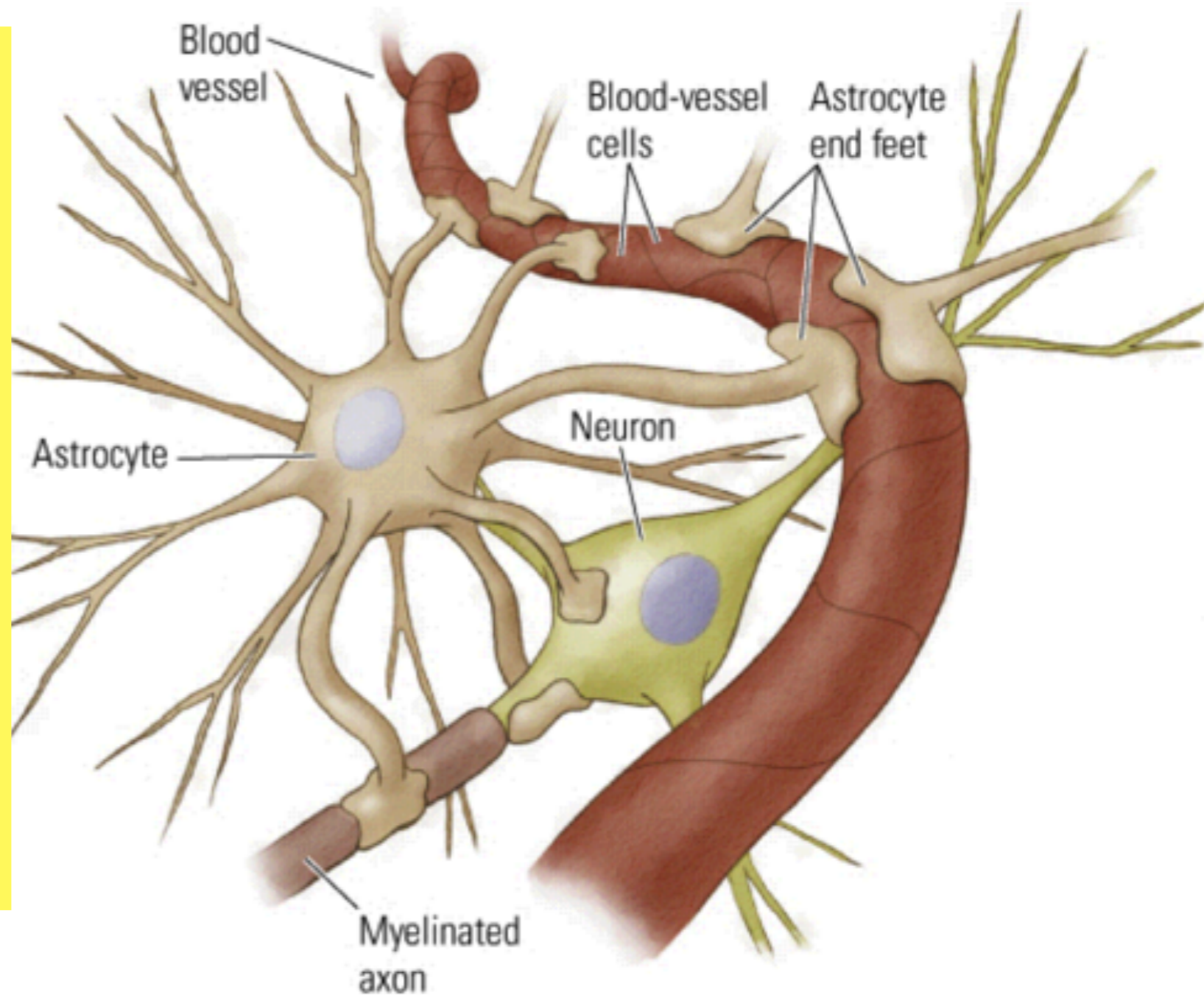


# Some Biology

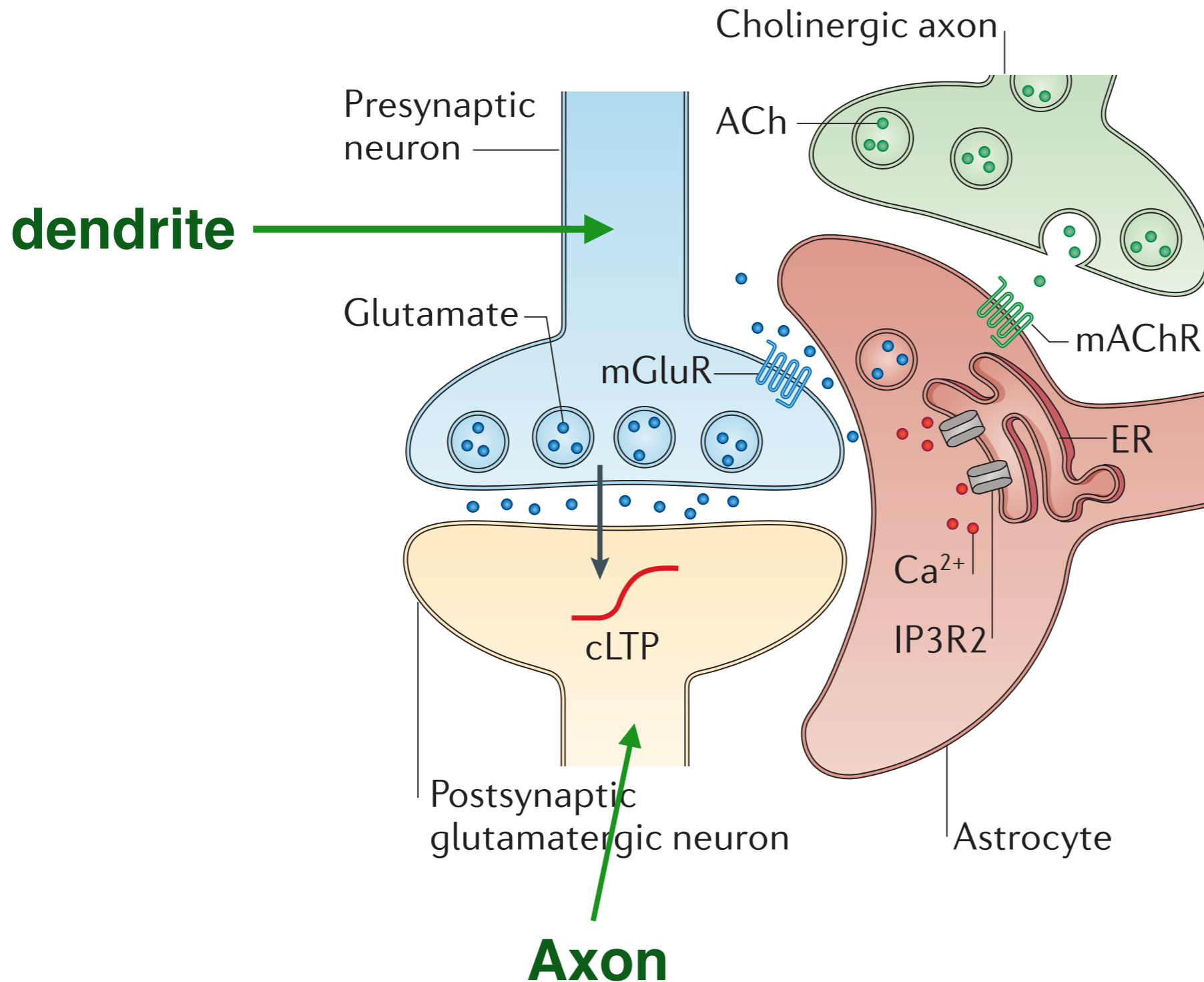
- Neurons
- Synapses
- Astrocytes
  - receptors, channels, ER
  - glutamate (neuro-transmitter)
  - IP3

# Astrocytes

- Astrocytes have many functions
  - provide nutrients to neurons
  - regulate calcium flow
  - play a role in various medical disorders (e.g. epilepsy)
  - modulate synaptic strength of neurons

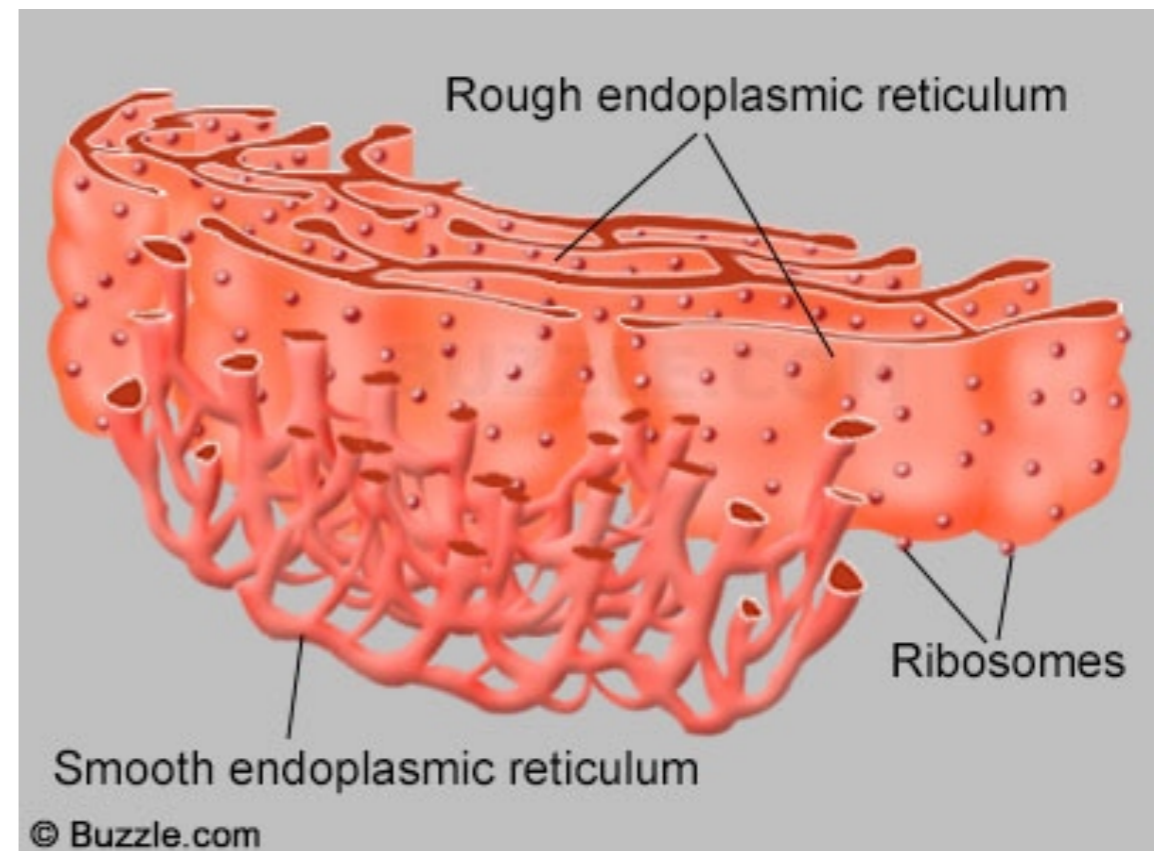


# Tripartite Configuration



# Endoplasmic Reticulum (in most cells)

An organelle is a watertight cellular compartment that acts as a store of calcium (among other things), which helps regulate calcium in the cytosol

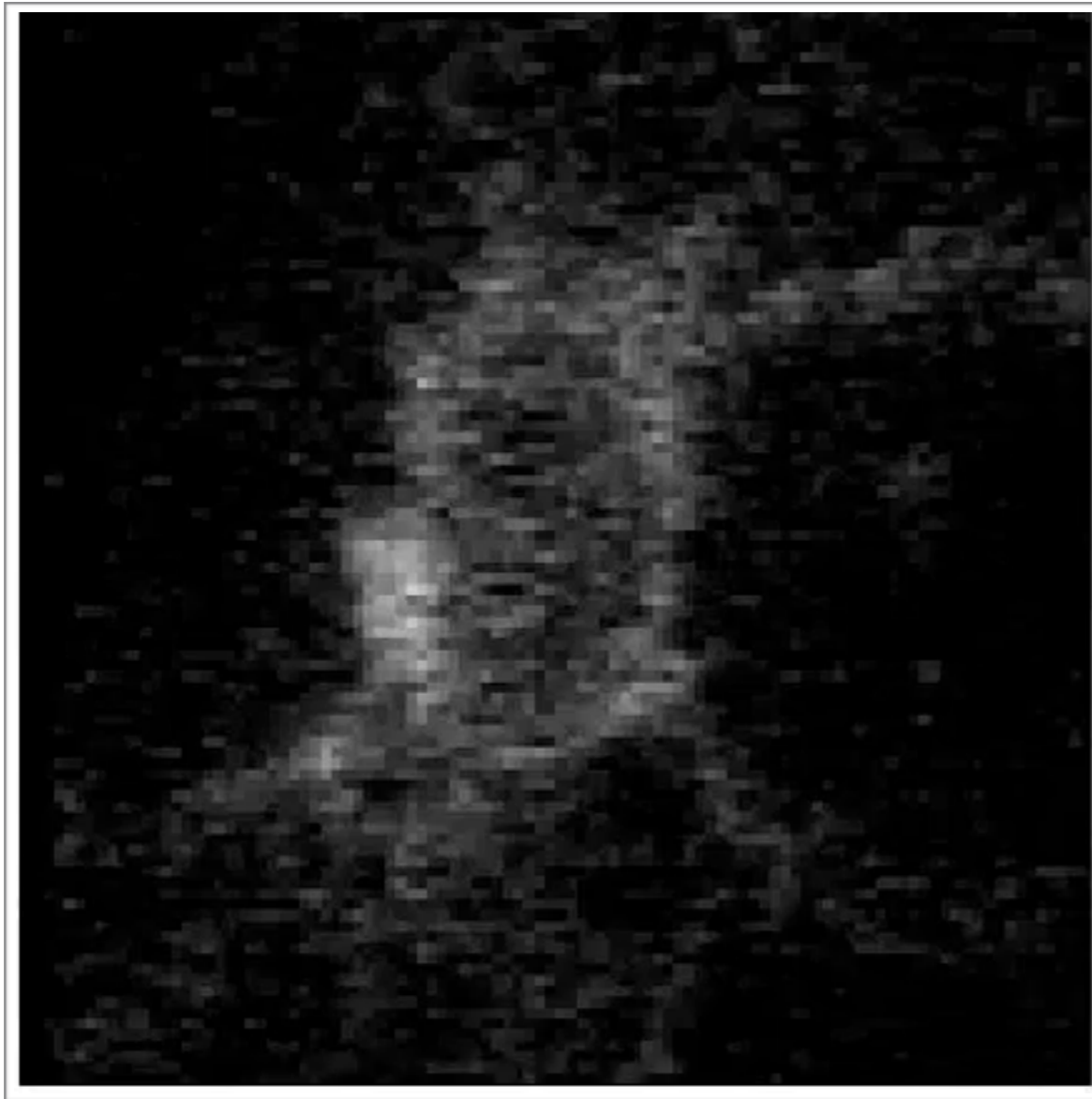


# Increased Temporal Resolution, 28Hz



# Astrocyte: 256x256

## 28Hz recording, raw data



Notice “spikes” in  
soma and processes

Origin of these  
“spikes” is non-  
electrical.

Time scales or orders  
of magnitude longer  
than spikes in neuron  
traces.

# Some Questions

- Does the soma integrate inputs from all processes and “spike” when the summed input reaches a threshold?
  - does the soma exhibit less activity than processes?
  - what is the origin and characteristics of the “spikes”?
  - are all processes equivalent in influencing soma activity?
  - must processes spike to influence soma activity?
- Do all processes influence each other’s activities or are they independently controlled by synaptic inputs?
- Do larger events have a larger spatial influence than smaller events?

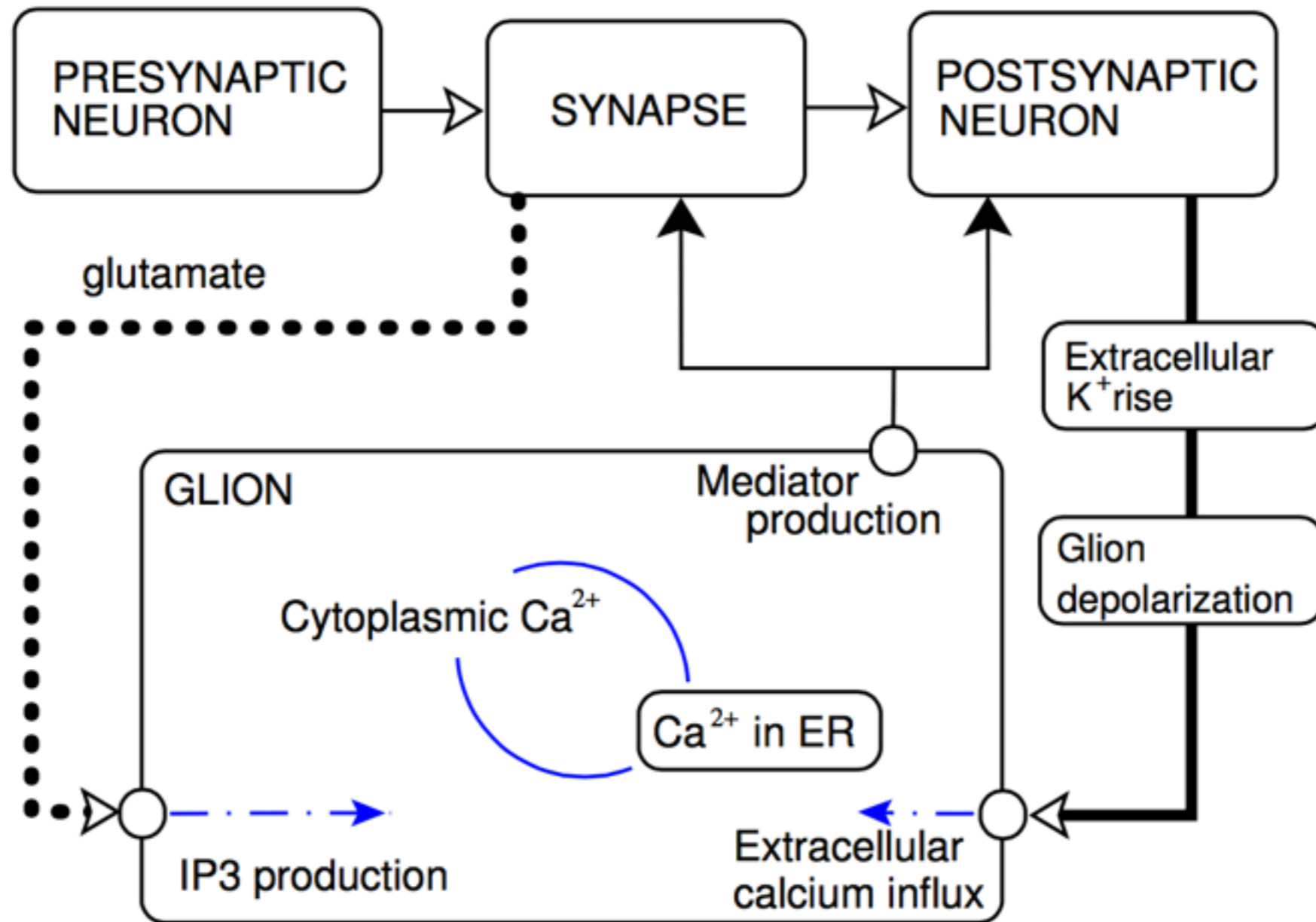


# Postnov Model (2007)

- Phenomenological
- Objective is to understand general characteristics, not quantitatively
- What kind of spiking can occur in the astrocyte, under what conditions
- Not modeling detailed biology

Biosystems. 2007 May-Jun;89(1-3):84-91. Epub 2006 Nov 12.  
Functional modeling of neural-gial interaction.  
Postno, Ryazanova, Sosnovtseva.

# Postnov (2007)



### Pre-synaptic neuron

$$\varepsilon_1 \frac{dv_1}{dt} = v_1 - \frac{v_1^3}{3} - w_1$$
$$\frac{dw_1}{dt} = v_1 + I_1 - I_{app},$$

### Post-synaptic neuron

$$\varepsilon_2 \frac{dv_2}{dt} = v_2 - \frac{v_2^3}{3} - w_2$$
$$\frac{dw_2}{dt} = w_2 + I_2 - I_{syn} - I_{glion},$$

### Synapse Activation

$$\tau_s \frac{dz}{dt} = (1 + \tanh(s_s(v_1 - h_s)))(1 - z) - \frac{z}{d_s},$$
$$I_{syn} = (k_s - \delta G_m)(z - z_0).$$

$$I_{glion} = \gamma G_m.$$

Strength of various interactions  
 $\alpha, \beta, \gamma$

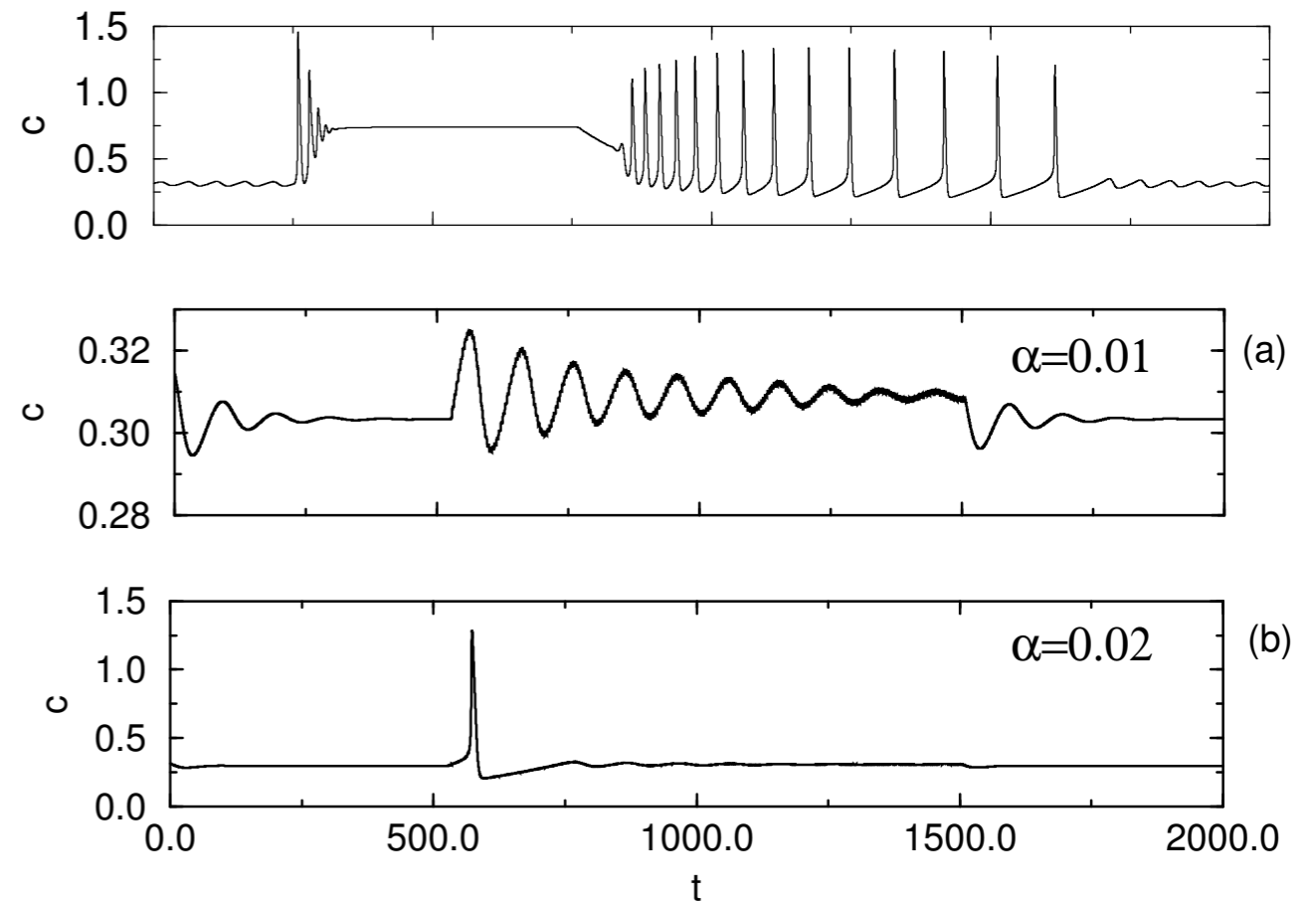
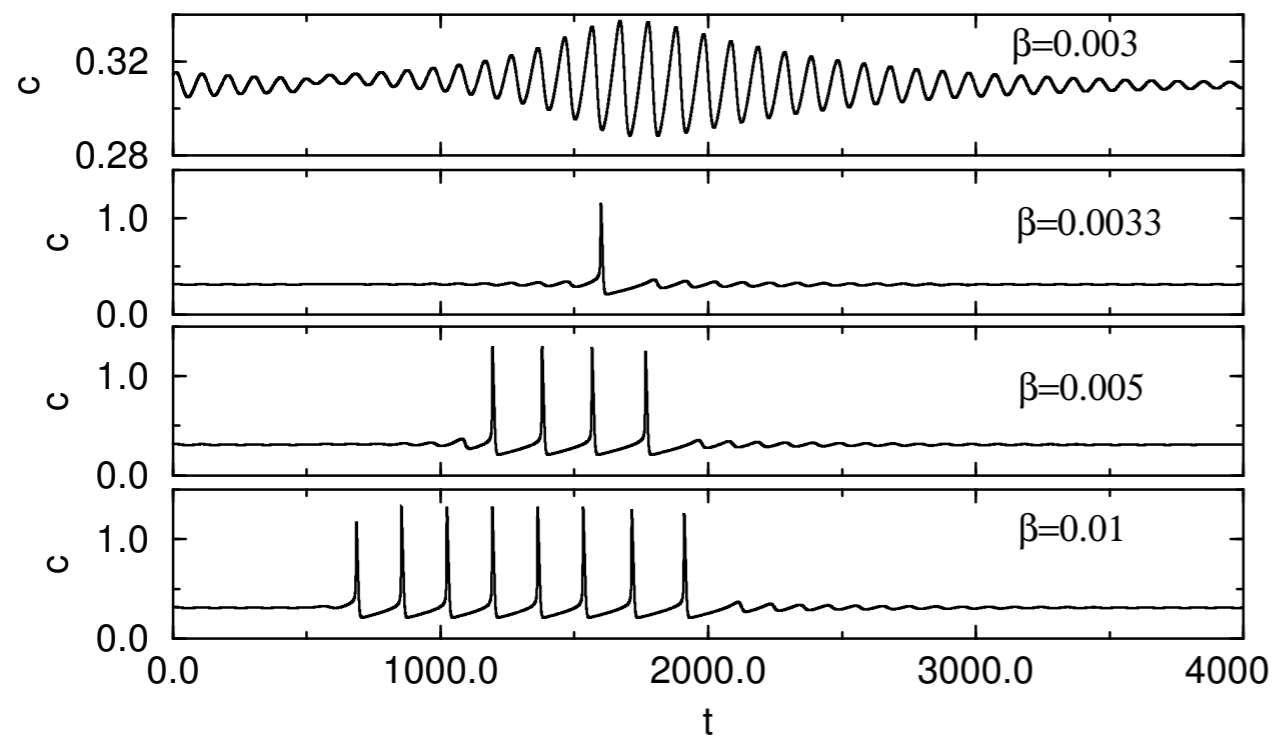
### Calcium Cytosol $\leftrightarrow$ ER

$$\tau_c \frac{dc}{dt} = -c - c_4 f(c, c_e) + (r + \alpha w_2 + \beta S_m),$$
$$\varepsilon_c \tau_c \frac{dc_e}{dt} = f(c, c_e).$$

### Sm: IP3, Gm: glion mediator

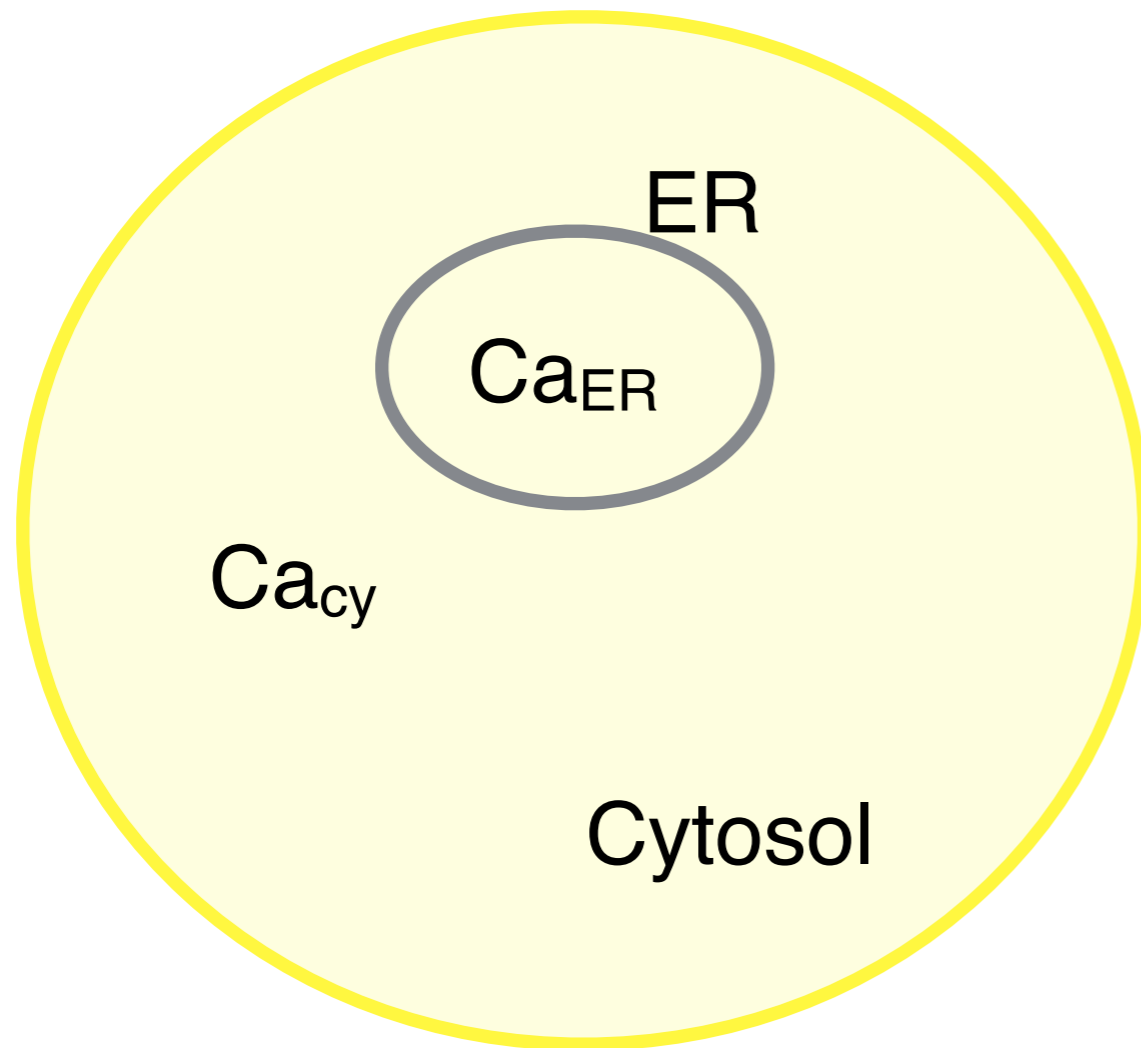
$$\tau_{S_m} \frac{dS_m}{dt} = (1 + \tanh(s_{S_m}(z - h_{S_m}))) \times$$
$$\times (1 - S_m) - \frac{S_m}{d_{S_m}},$$
$$\varepsilon_c \tau_{G_m} \frac{dG_m}{dt} = (1 + \tanh(s_{G_m}(c - h_{G_m}))) \times$$
$$\times (1 - G_m) - \frac{G_m}{d_{G_m}},$$

# Some Results for Calcium Postnov (2007)



Just shows the diversity of calcium spiking inside the Soma  
Spiking is defined as width over spike-interval very very small

# Single Point Astrocyte



## Variables

Ca: calcium in cytosol

Ca<sub>ER</sub>: calcium in ER

## Parameters

Secondary messenger: IP<sub>3</sub>

Neurotransmitter: Glutamate

# Compartmental model

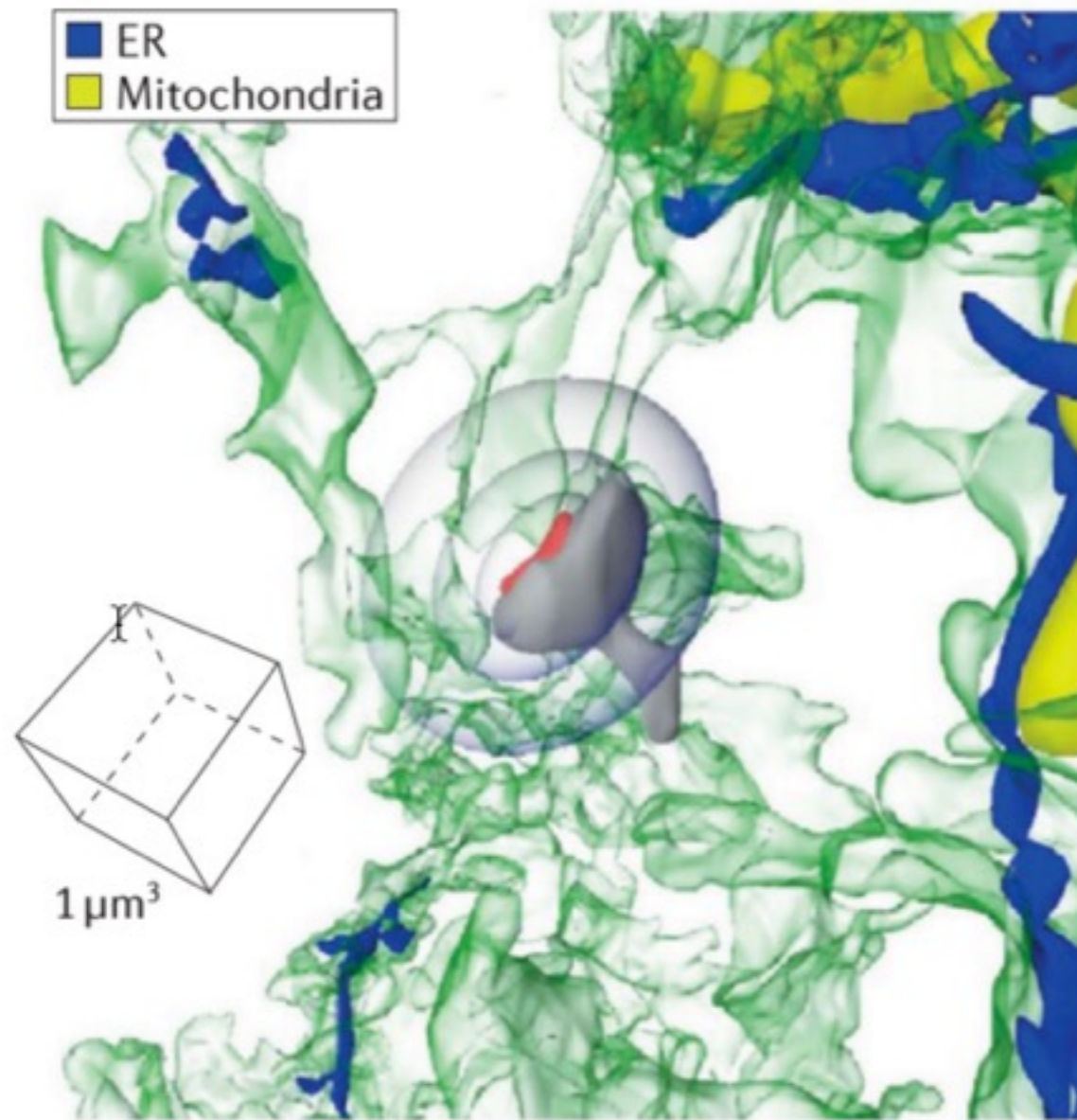
- Treat separately
  - each process
  - the soma

$r$ : effect of neurons on astrocyte

$d_c$ : Diffusion coef. in Cytosol

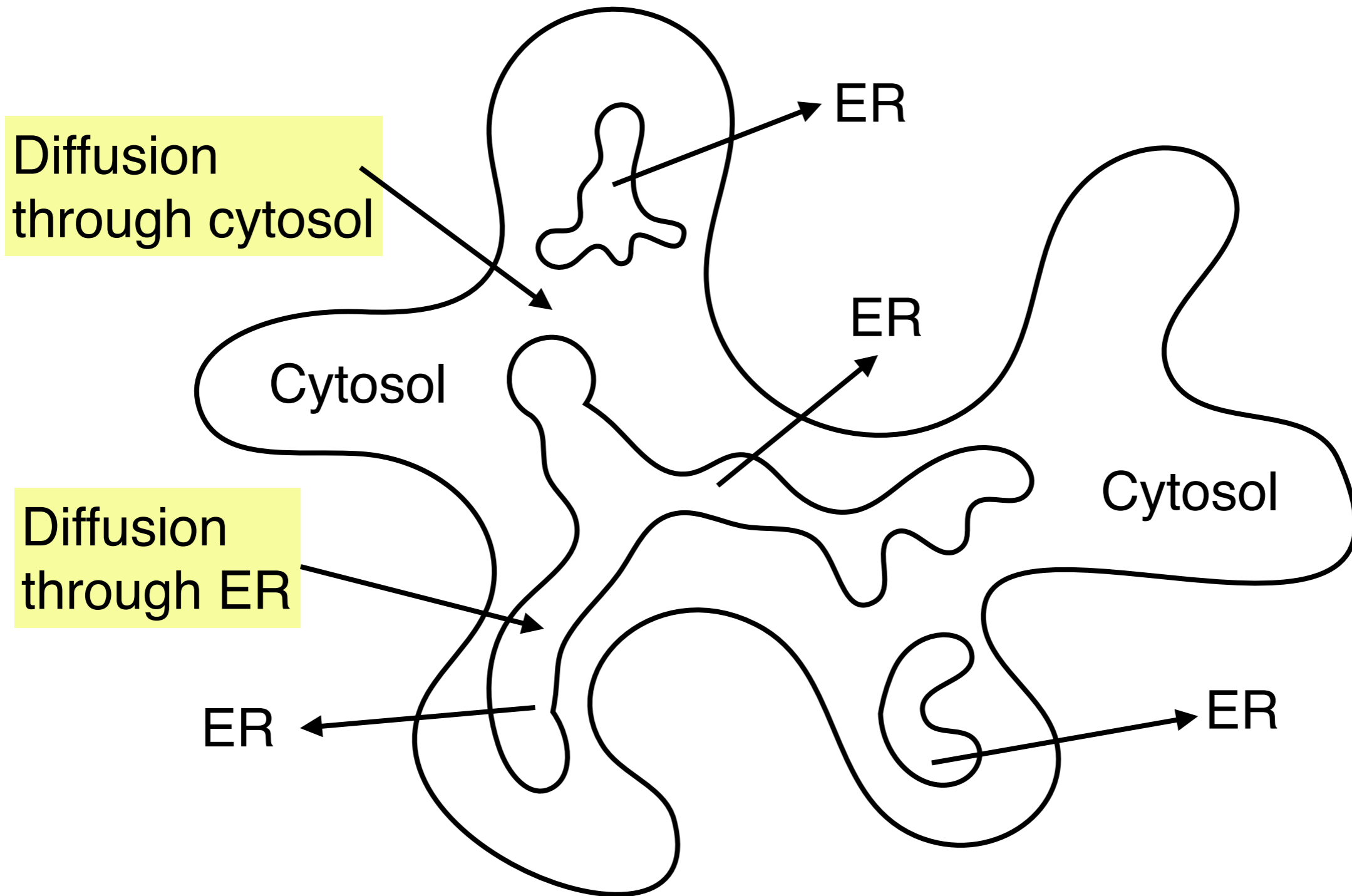
$d_{er}$ : Diffusion coef. in ER

# Endoplasmic Reticulum of an Astrocyte



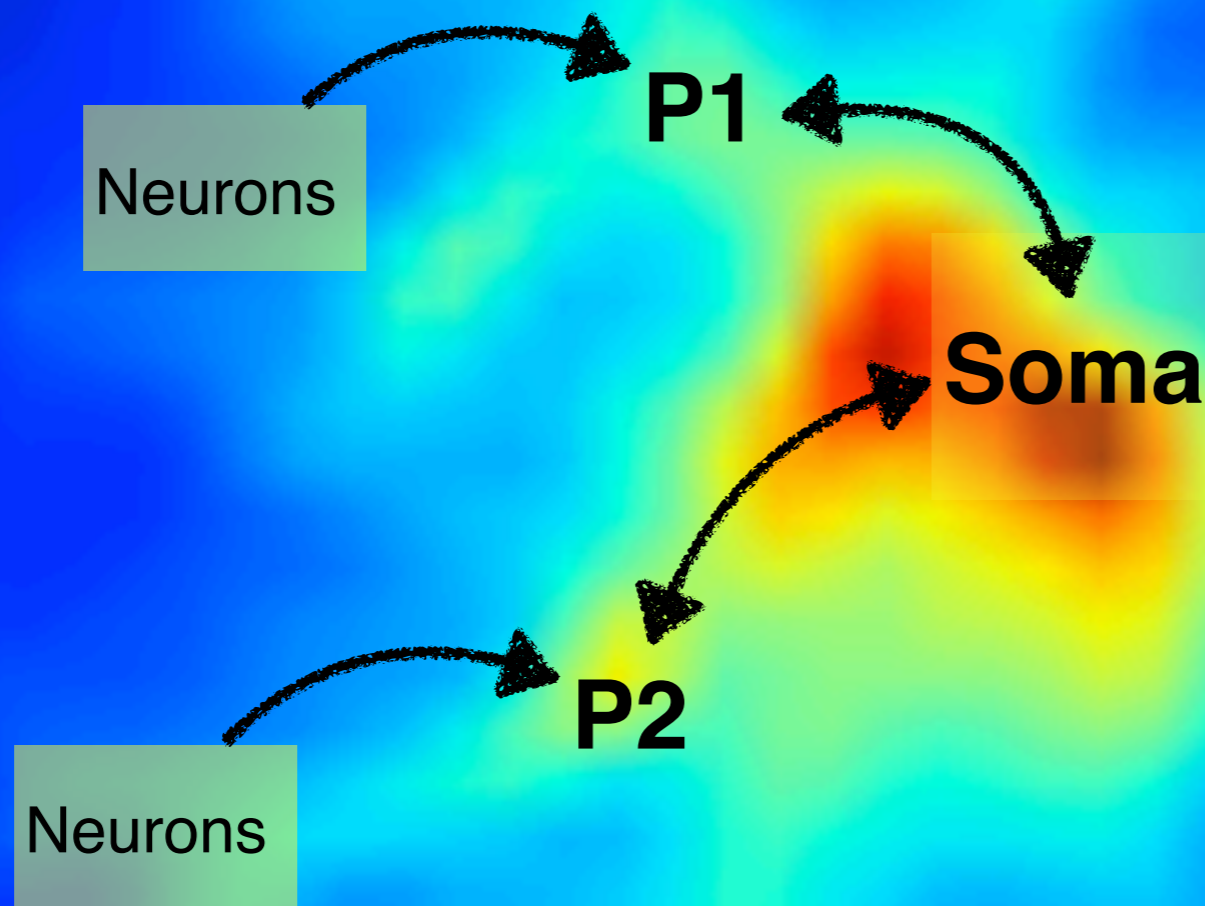
Singly connected entity?  
Multiply connected?

# How Are The ER's Connected?



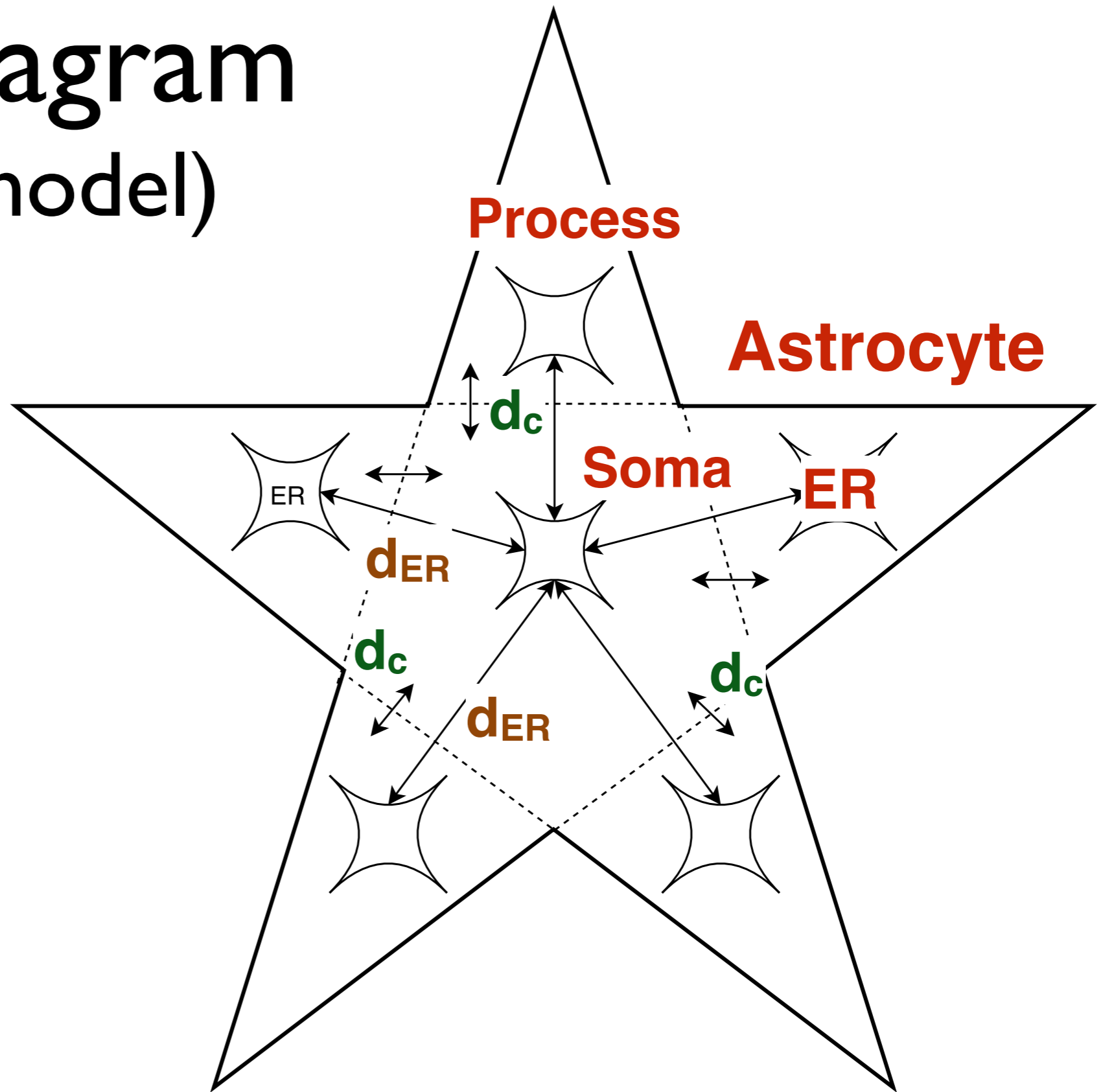


# Idea Behind Our Model



# Model Diagram (N point model)

**N processes**



# Our Model Based On Postnov (neurons modeled through $r_{Amp}(t)$ )

$$\tau_p \frac{dCa_{p_i}}{dt} = \overbrace{r_L + r_{Amp}} + \cancel{r + \alpha W_{post_i} + \beta S m_i} - c_4 * f(Ca_{p_i}, Ca_{er_i}) + \underbrace{d_c(Ca_s - Ca_{p_i})}_{\text{Diffusion}} - Ca_{p_i}$$

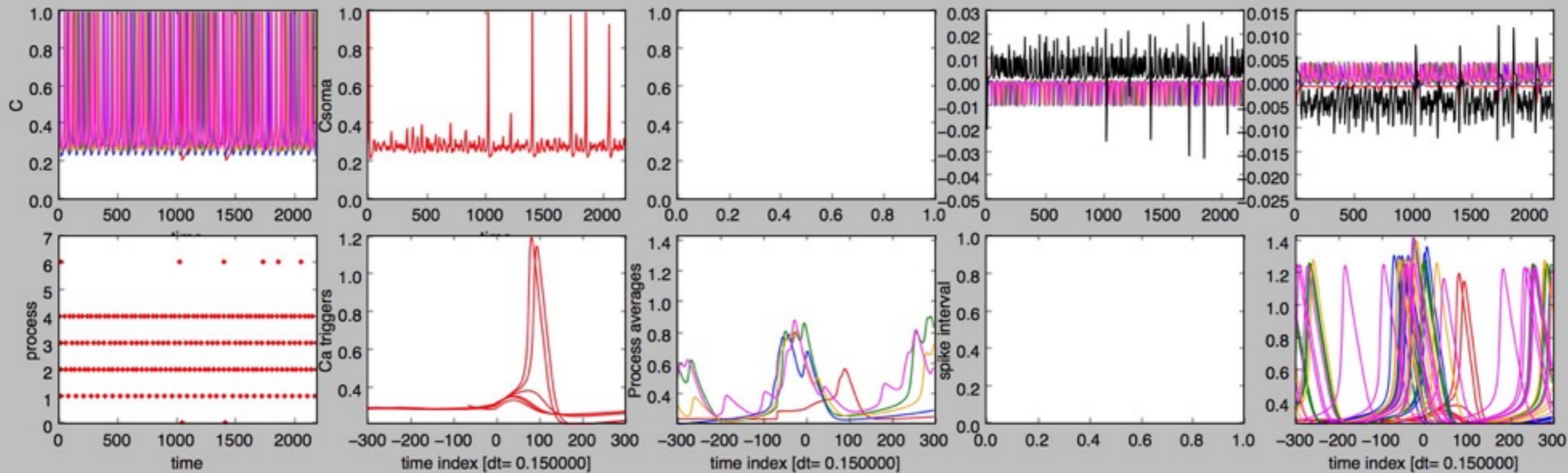
$$\epsilon_c \tau_c \frac{dCa_{er_i}}{dt} = f(Ca_{p_i}, Ca_{er_i}) + \underbrace{d_{er}(Ca_{er_s} - Ca_{er_i})}_{\text{Diffusion}}$$

$$\tau_c \frac{dCa_s}{dt} = -Ca_s - c_4 * f(Ca_s, Ca_e) + \underbrace{\left( r + \sum_{i=1}^n d_c(Ca_{p_i} - Ca_s) \right)}_{\text{Diffusion}}$$

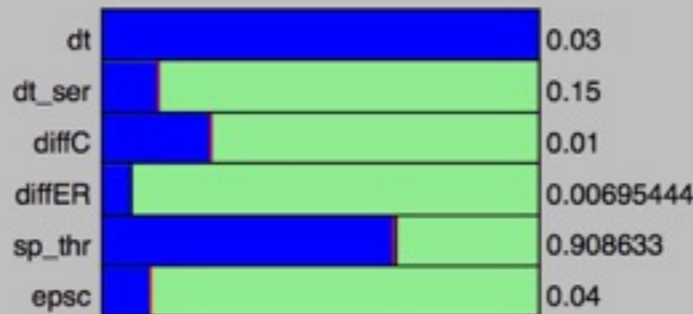
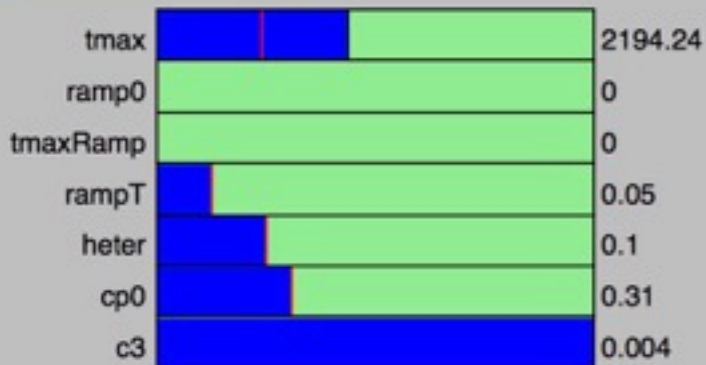
$$\epsilon_c \tau_c \frac{dCa_{er_s}}{dt} = f(Ca_s, Ca_{er_s}) + \underbrace{\sum_{i=1}^n d_{er}(Ca_{er_i} - Ca_{er_s})}_{\text{Diffusion}}$$

**Diffusion**

$$f(c, c_e) = c_1 \frac{c^2}{1 + c^2} - \left( \frac{c_e^2}{1 + c_e^2} \right) \left( \frac{c^4}{c_2^4 + c^4} \right) - c_3 c_e$$



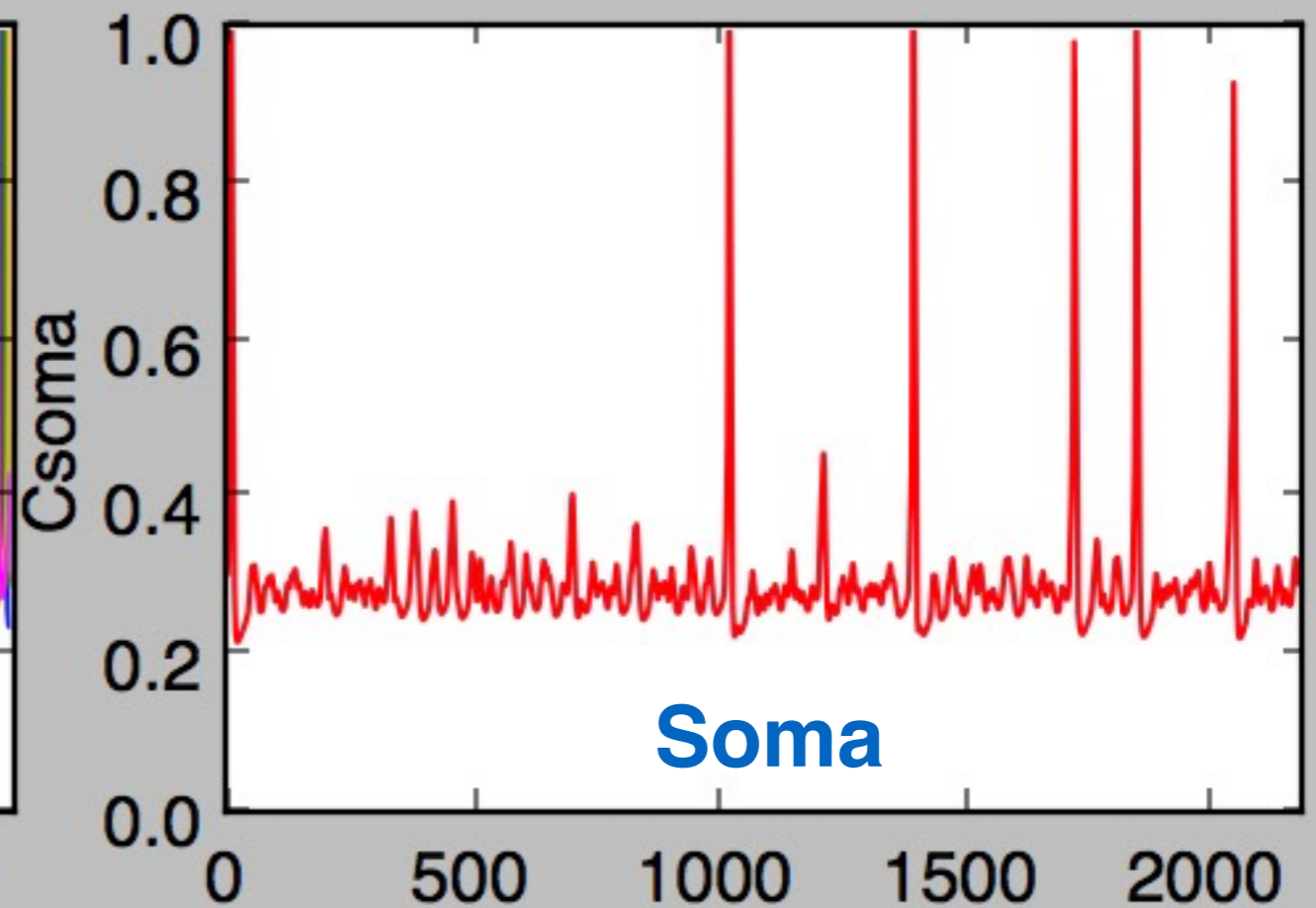
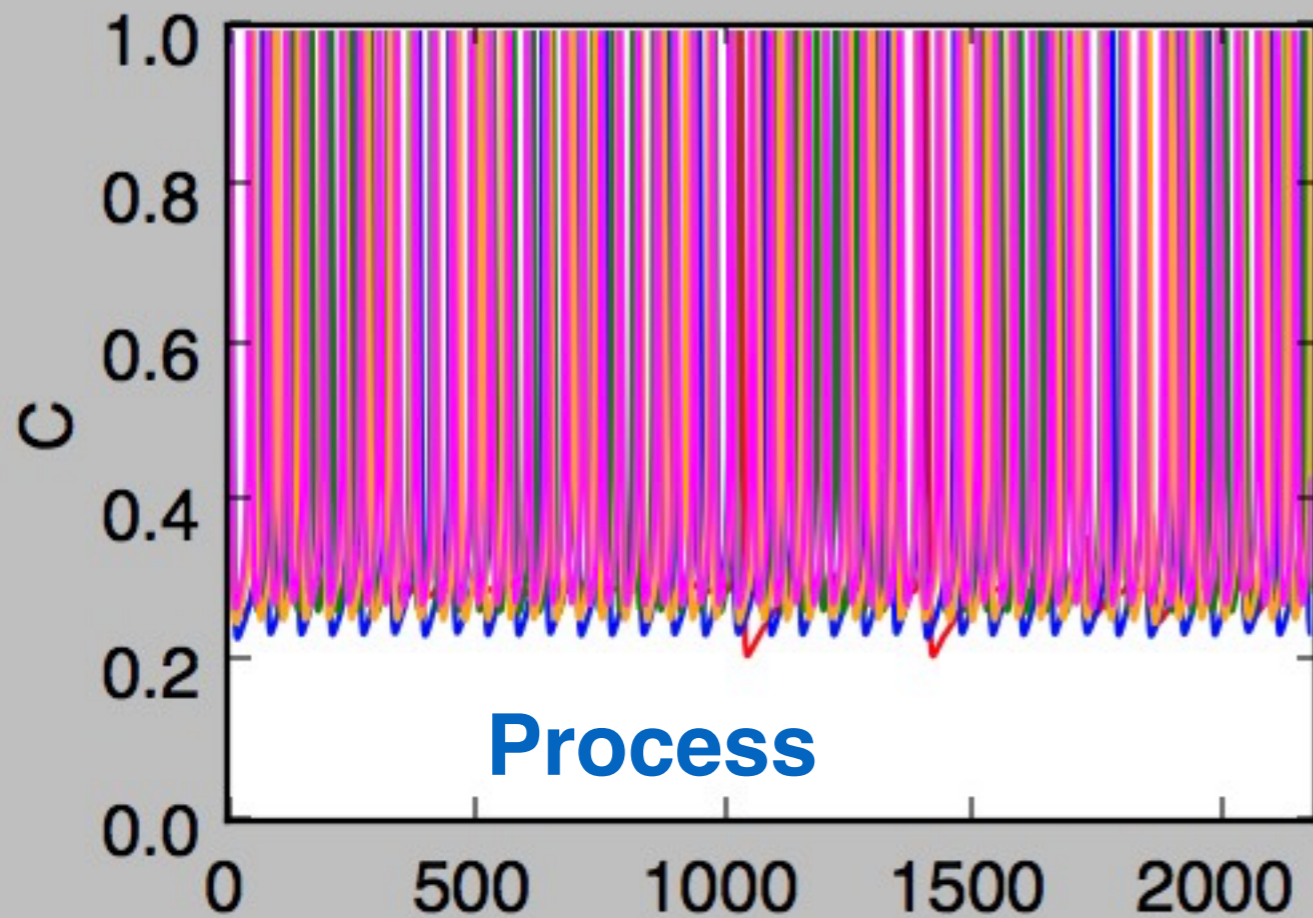
toggle



time win 300

Python (GUI) +  
C++ Simulation  
(5 processes)

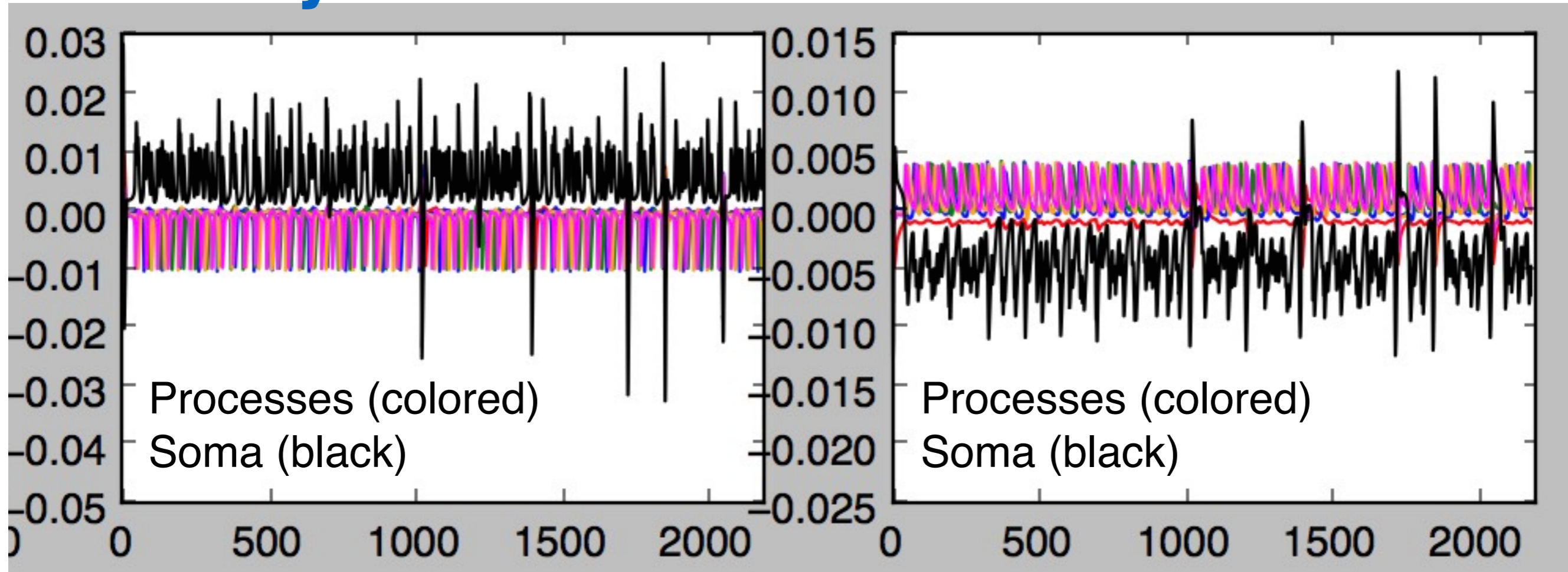
# Calcium In The Process And The Soma



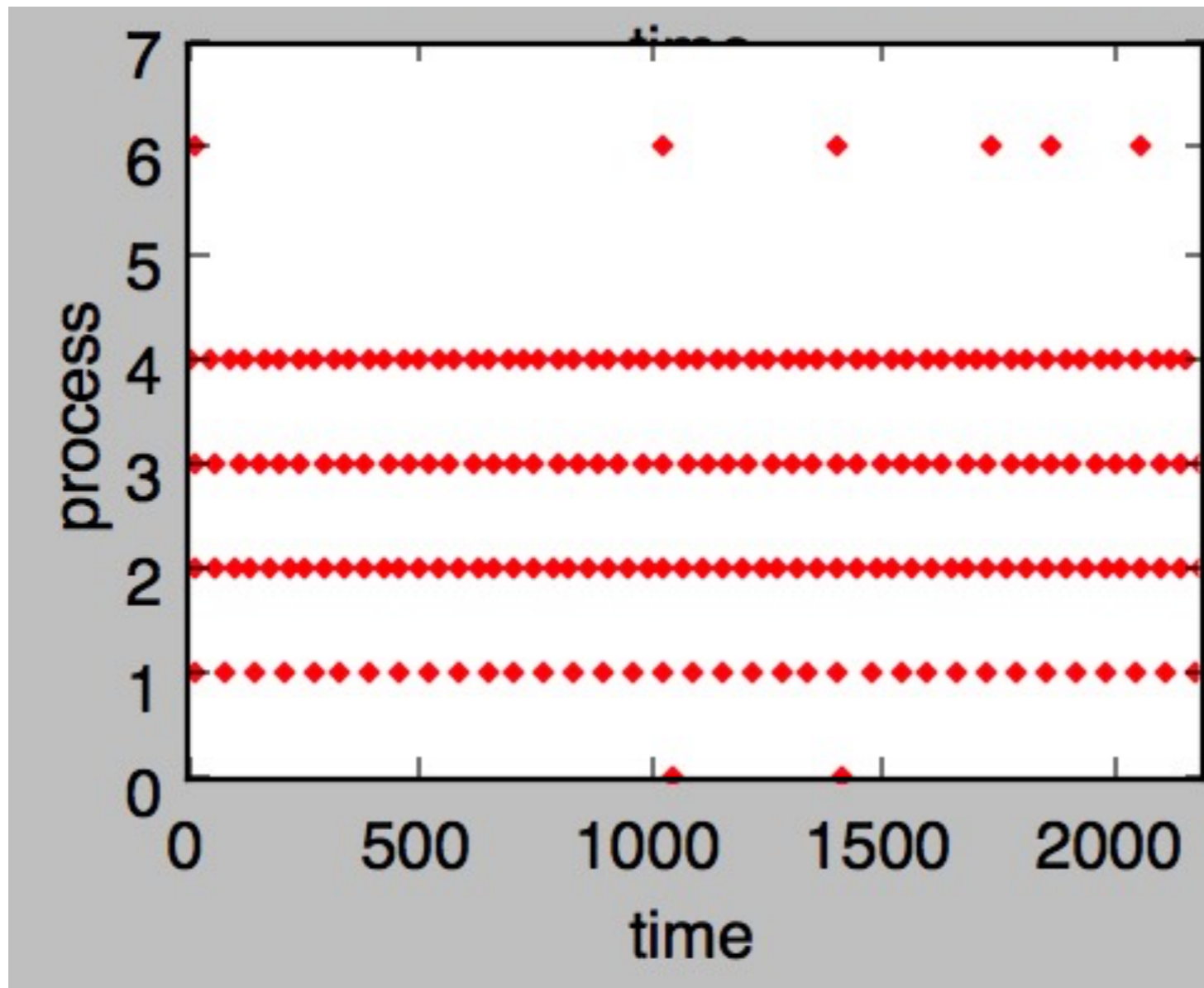
# Diffusion terms

Cytosol

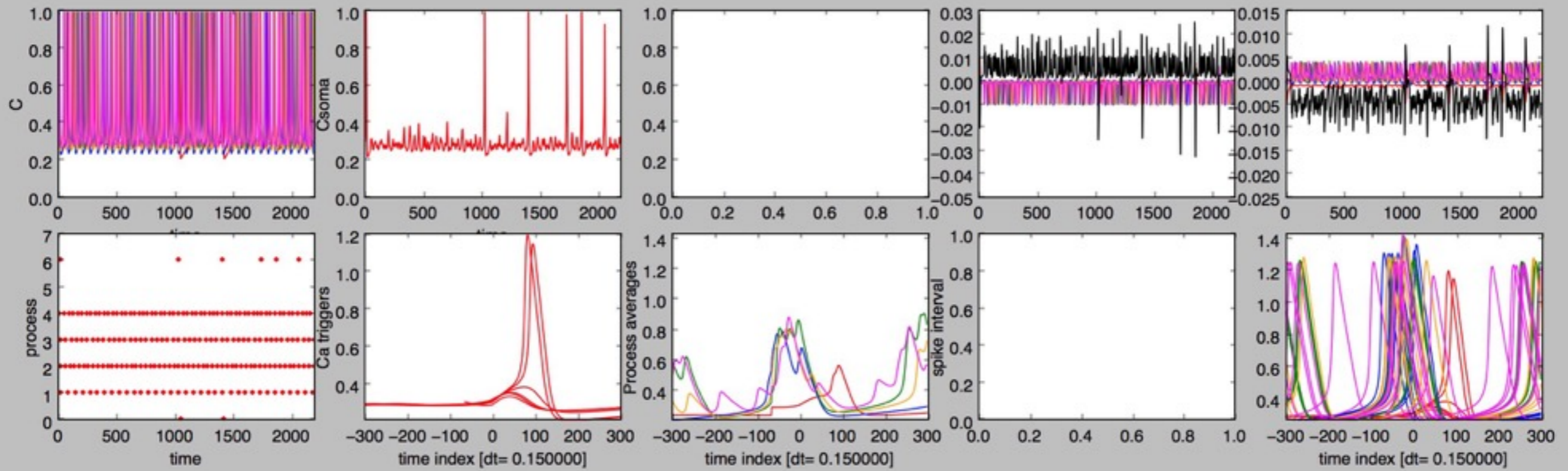
ER



# Raster Plot

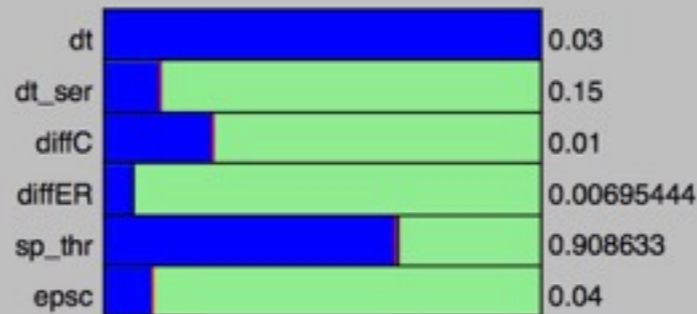
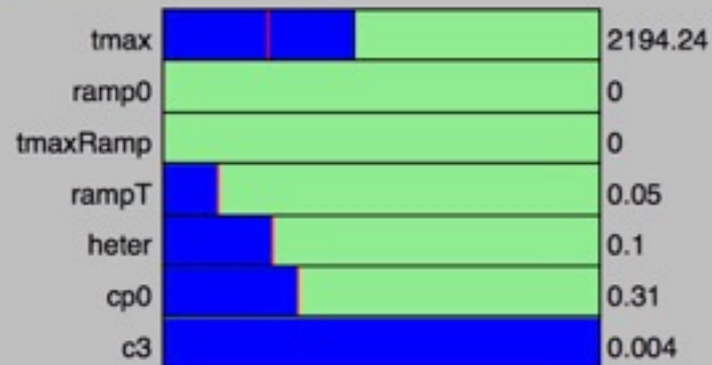


Record each spike  
when the signal is  
above the threshold



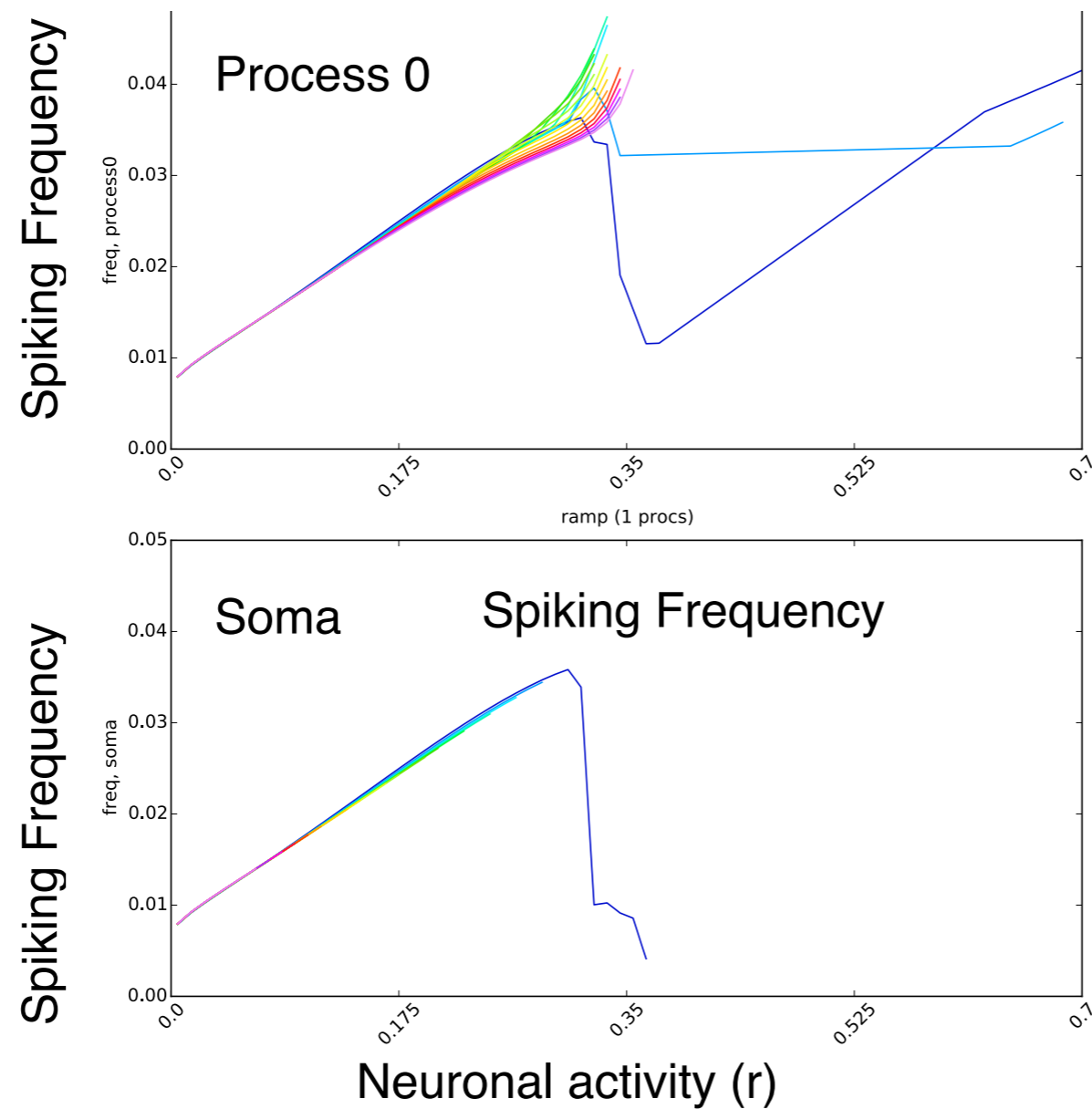
# Output from C++/Python Simulation Code

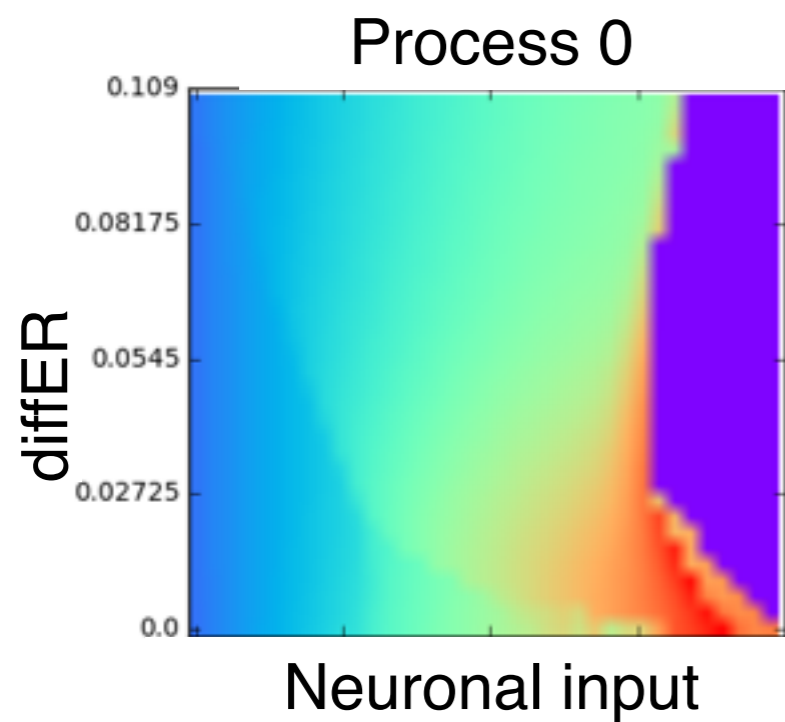
toggle



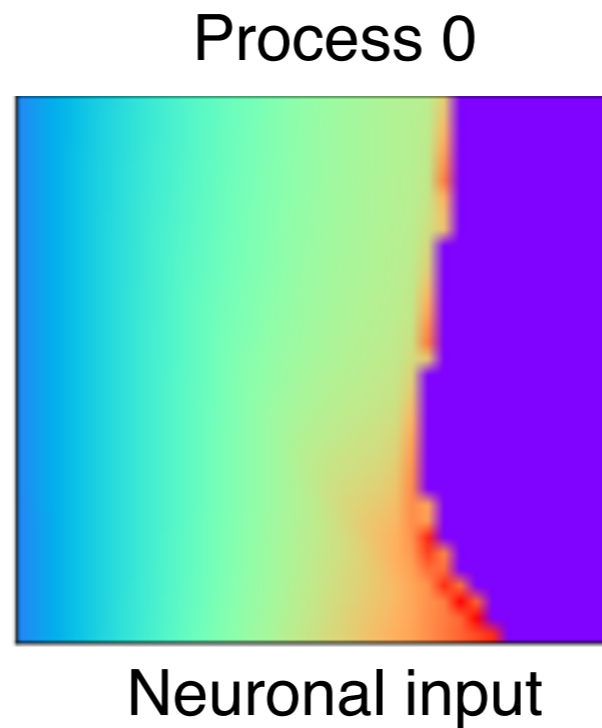


# Spiking Frequency as a function of neural activity for different values of $d_{er}$ (Diff ER)

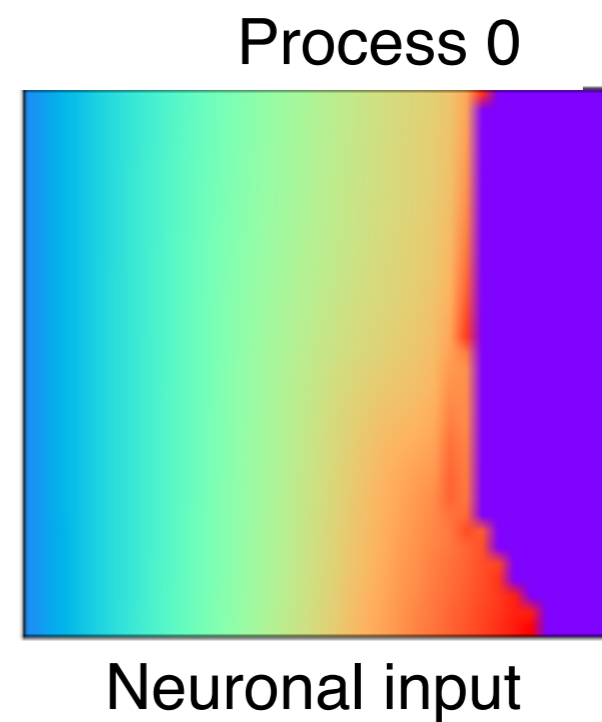




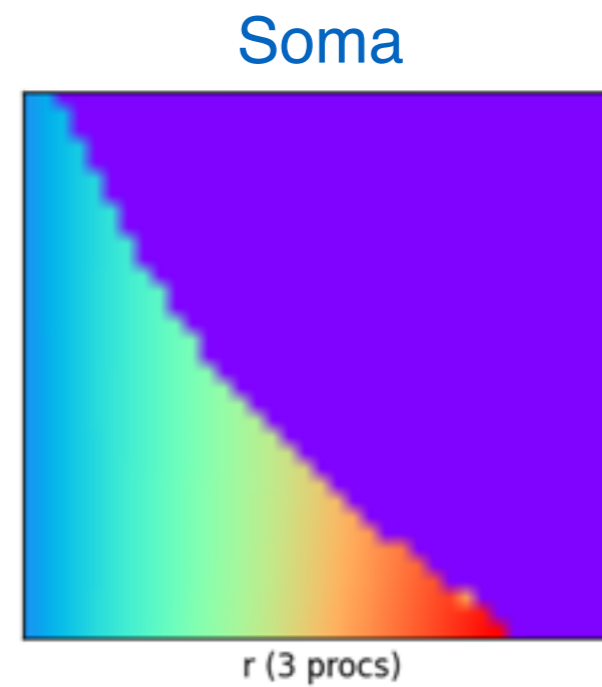
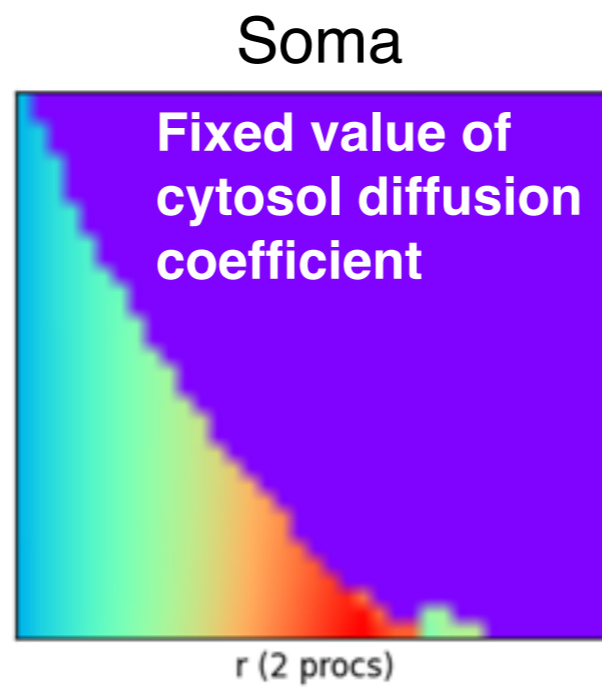
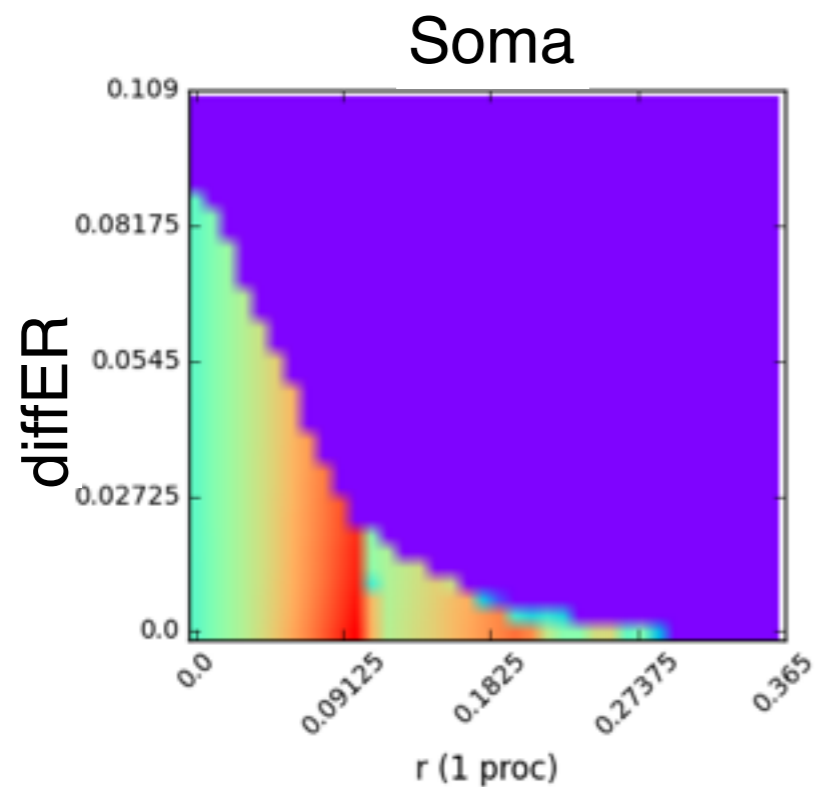
**1 process**



**2 processes**

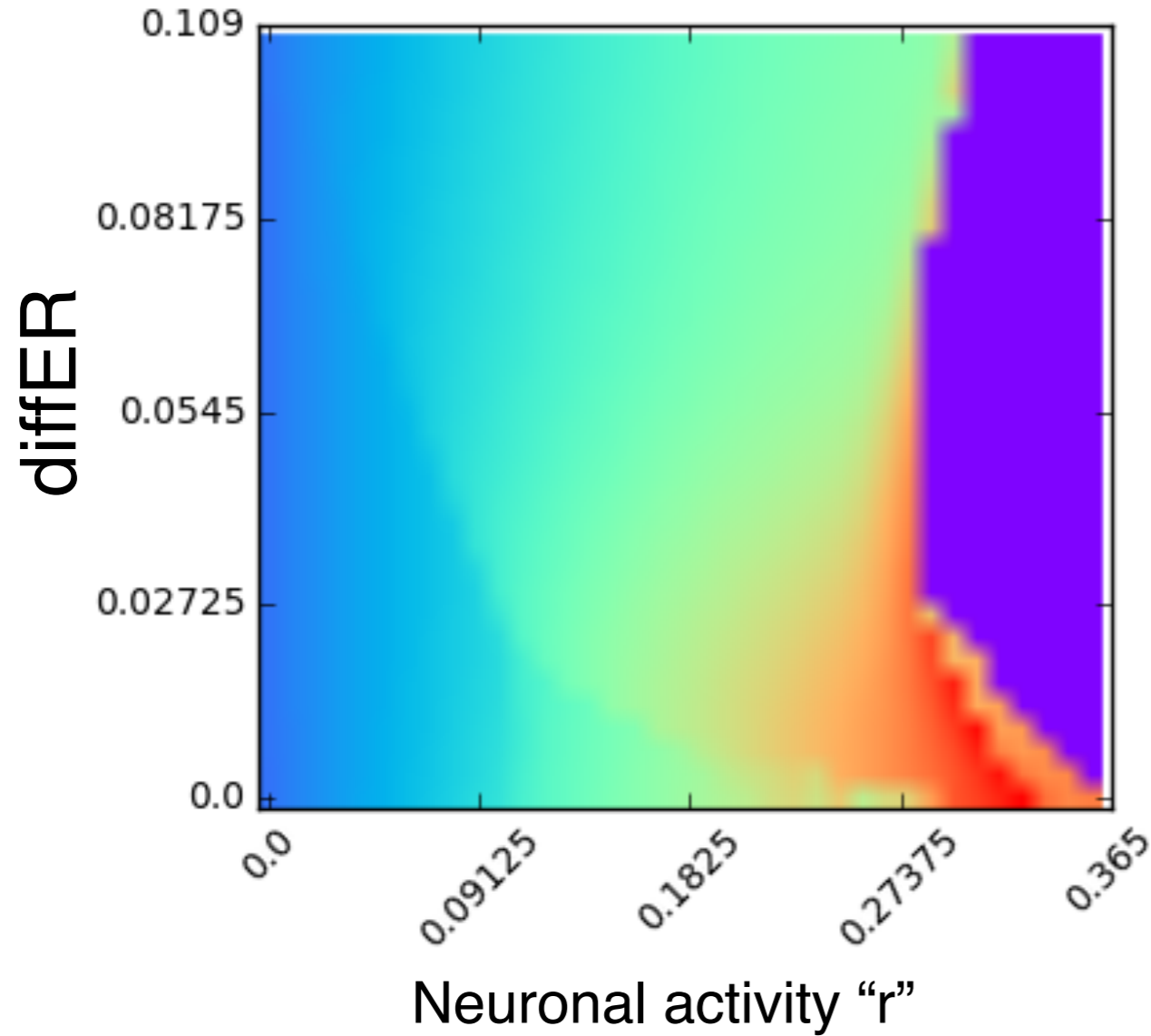


**3 processes**

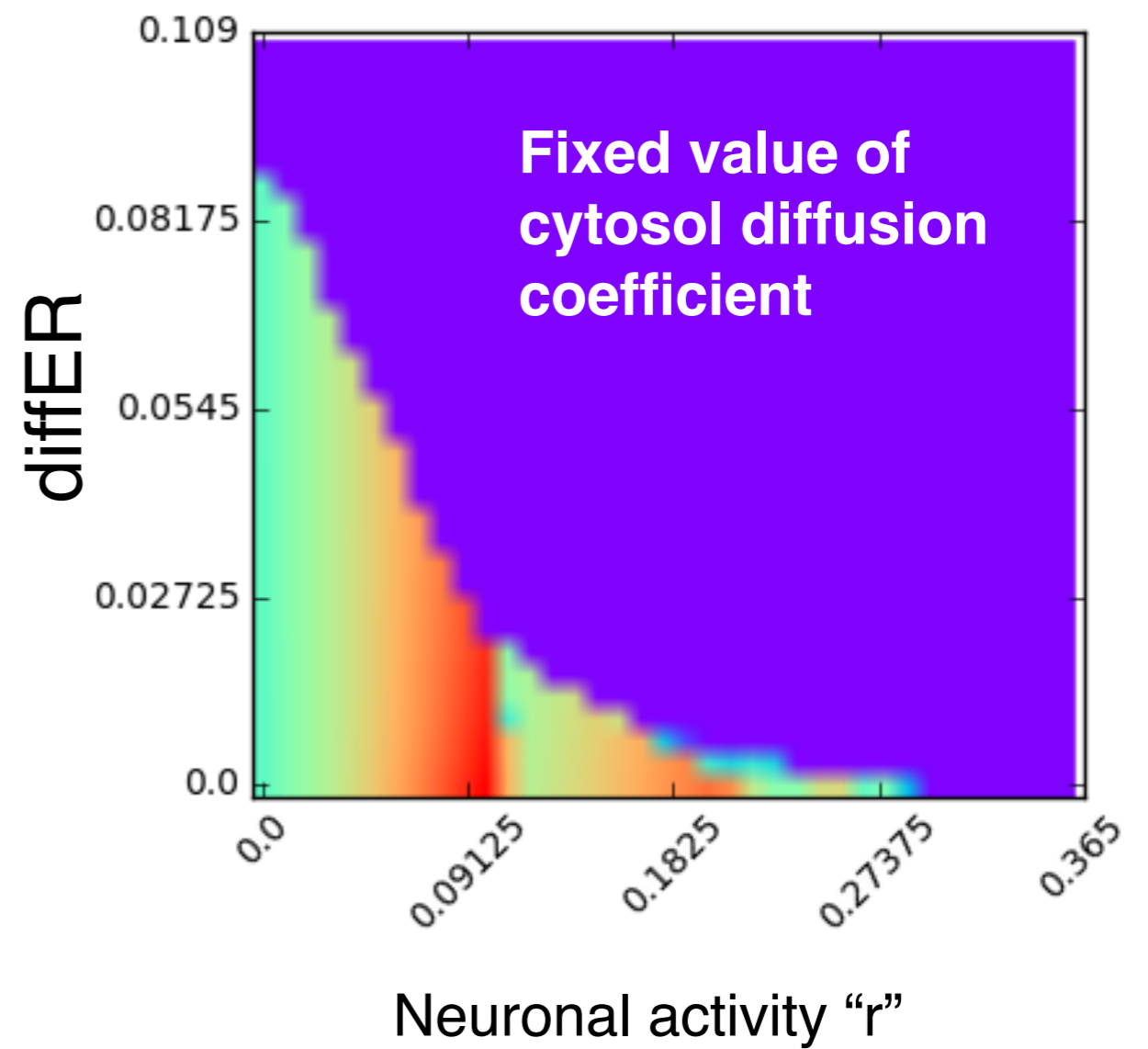


# One process + Soma

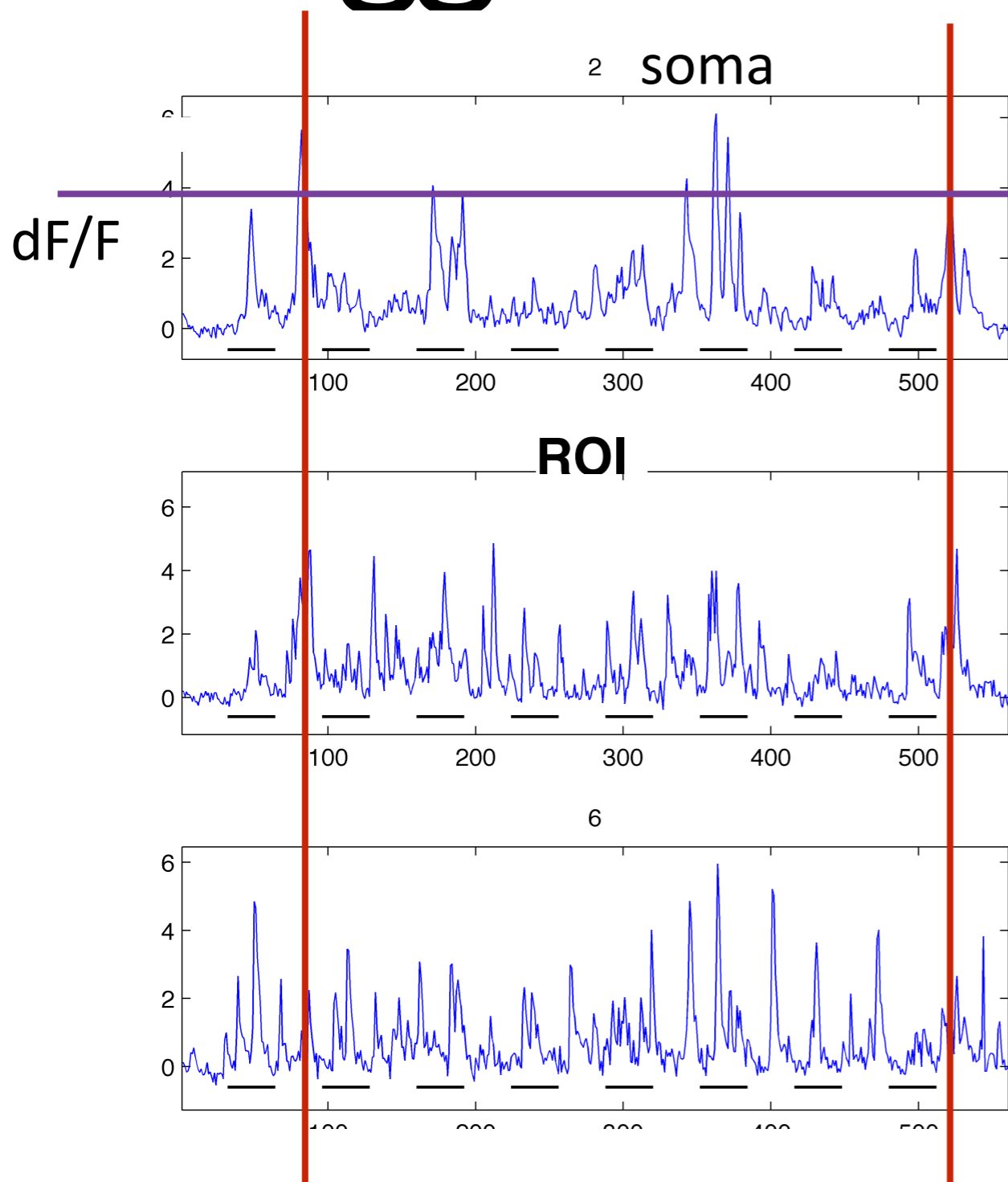
Process 0



Soma

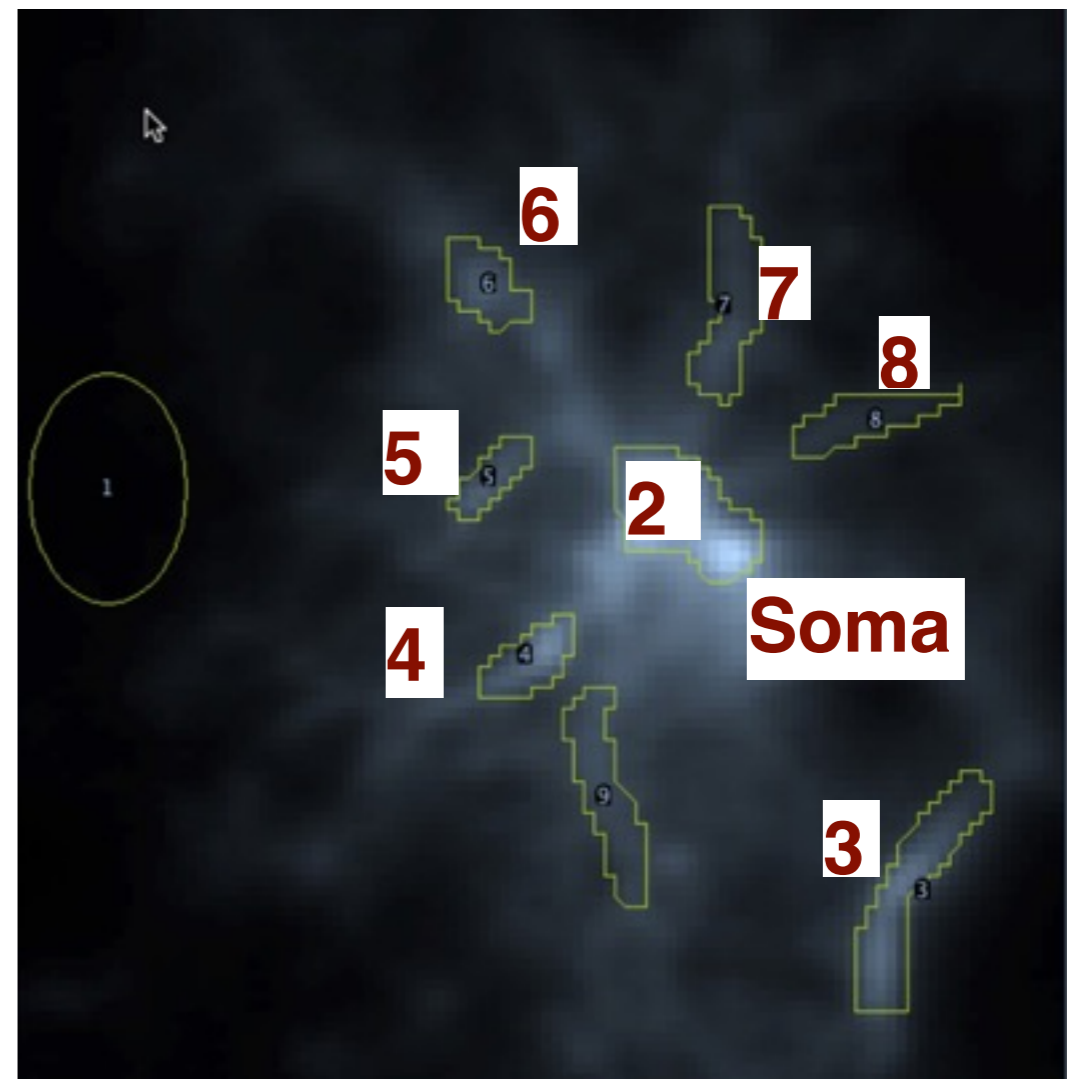


# Trigger-Based Averaging



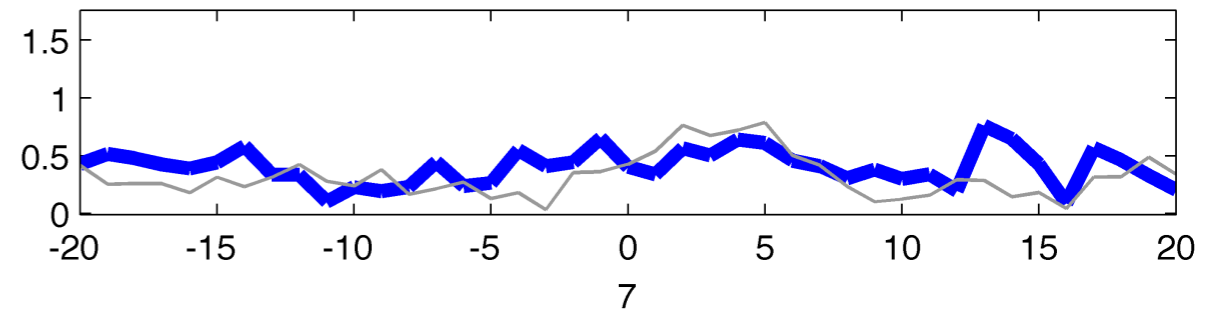
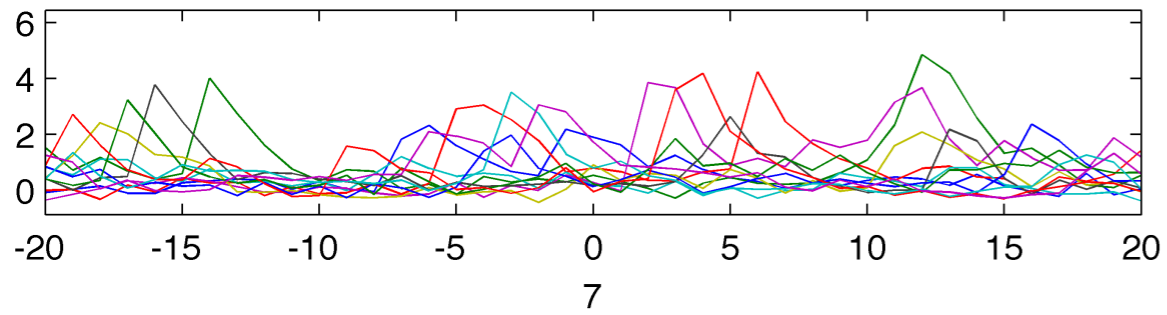
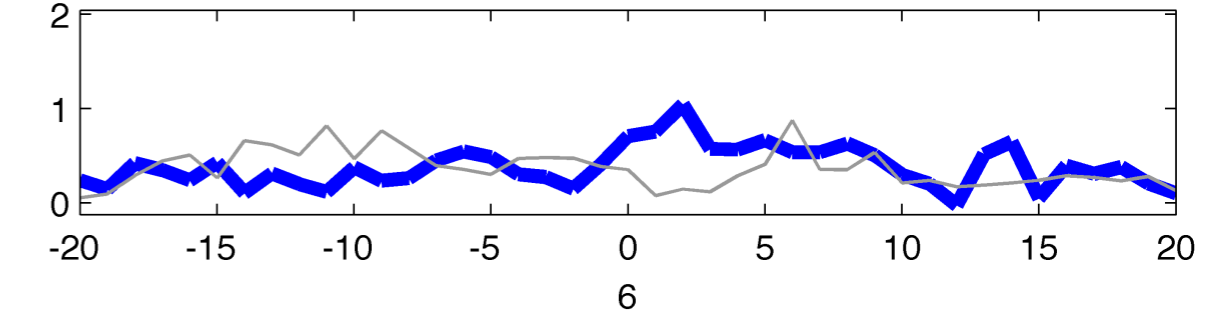
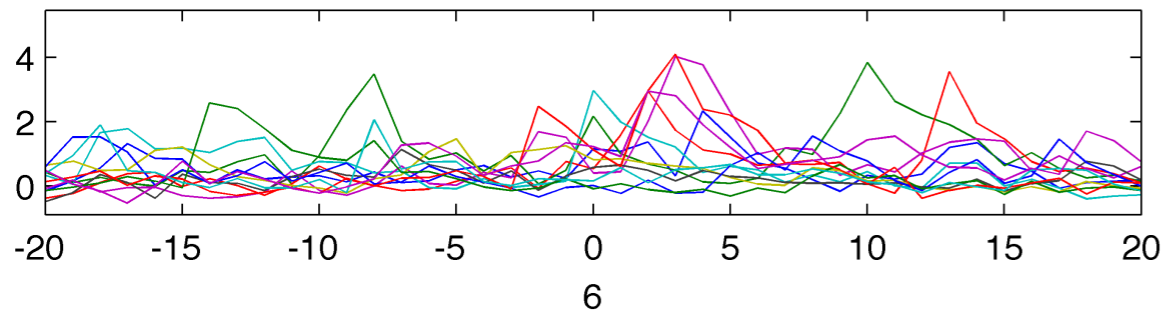
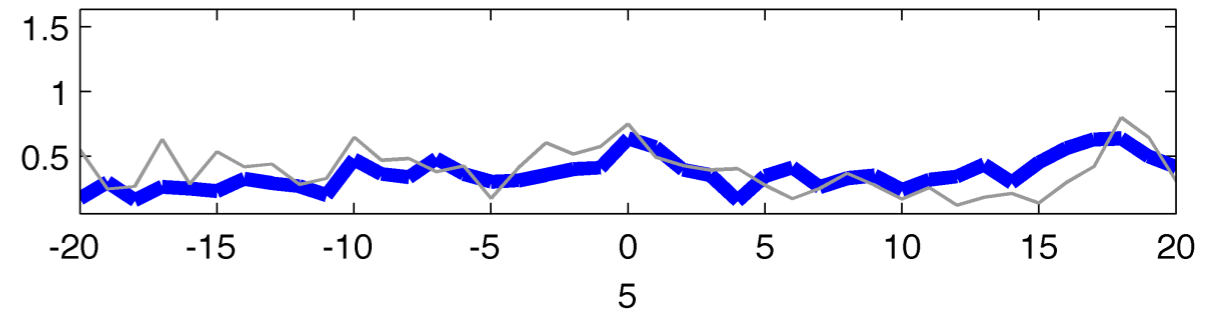
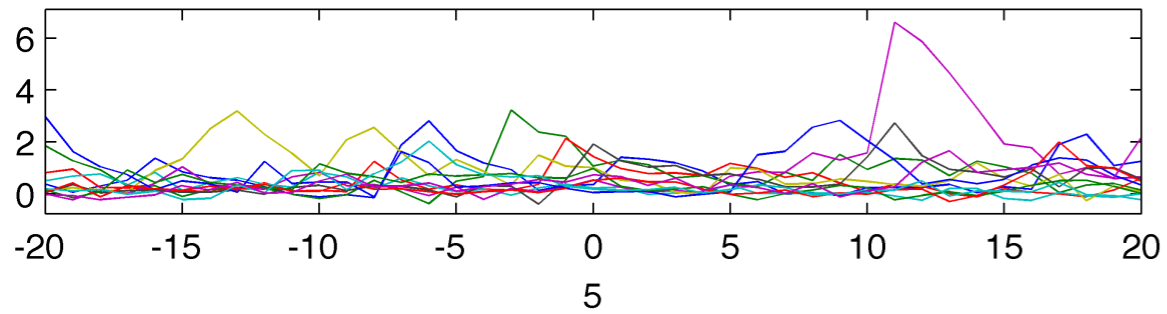
Arbitrary threshold

Spikes in soma



# Trigger-Based Averages

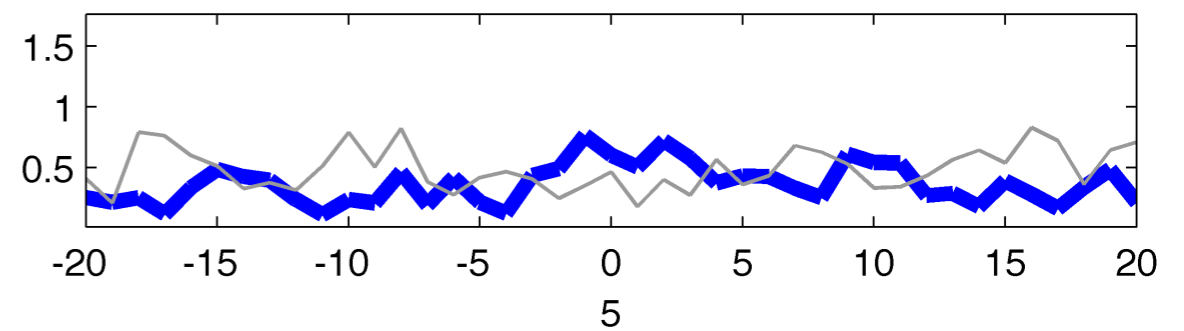
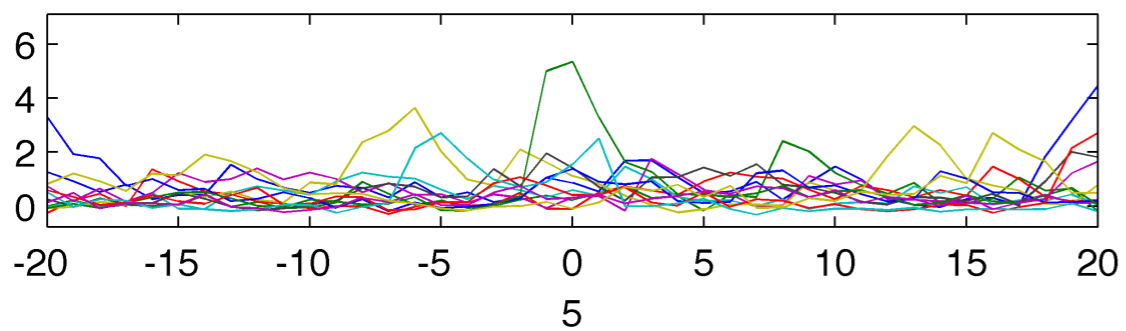
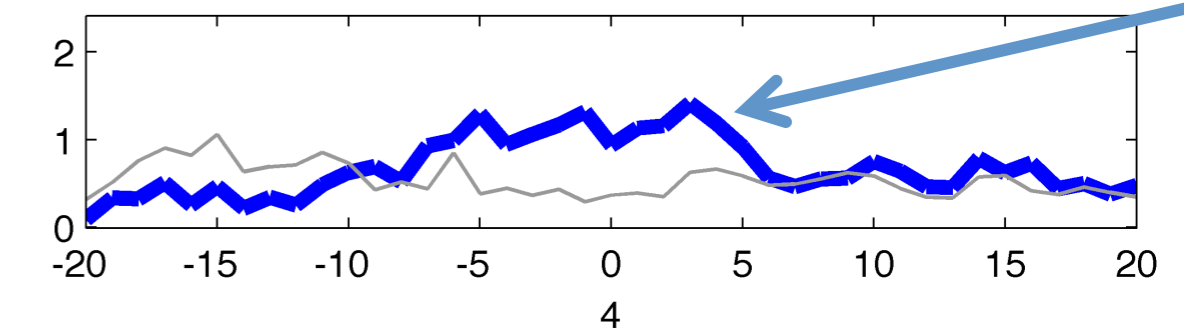
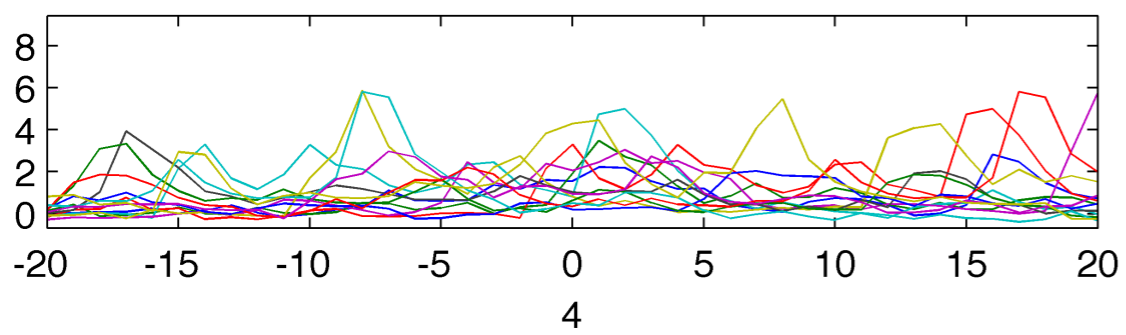
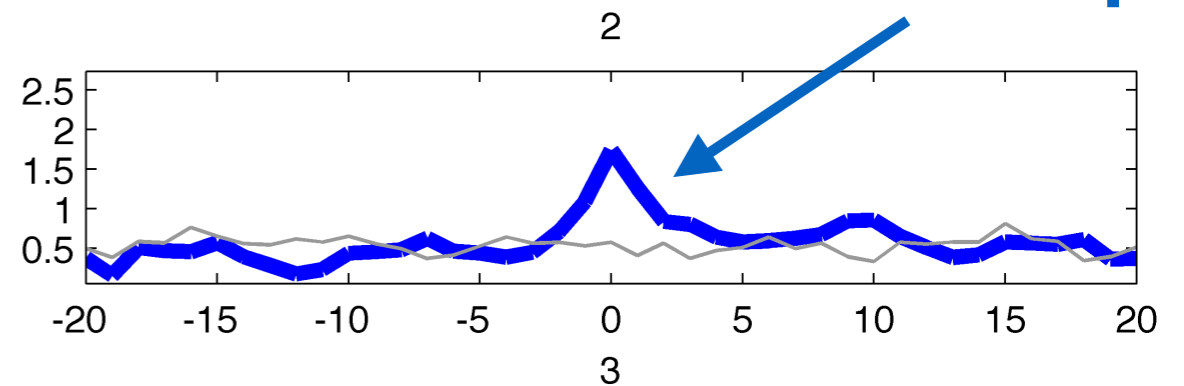
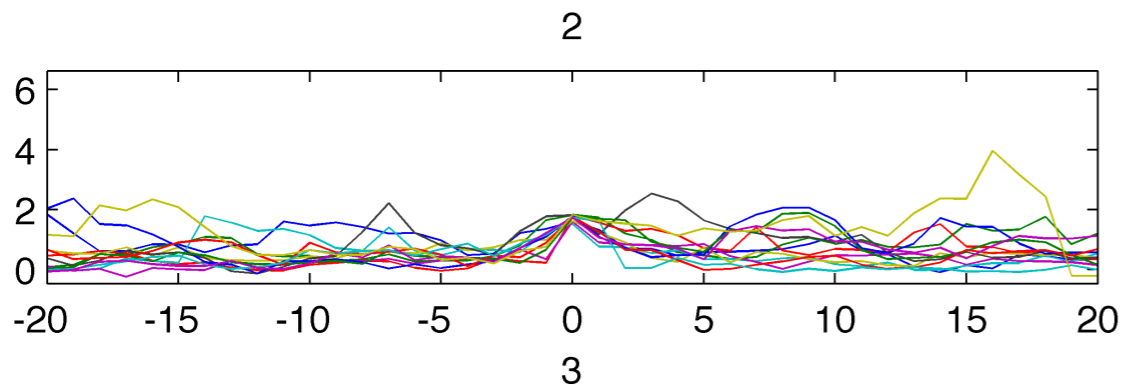
Threshold Amplitude: 1.25 - 1.5



# Trigger-Based Averages

Threshold Amplitude: 1.5 - 2.0

STA bump

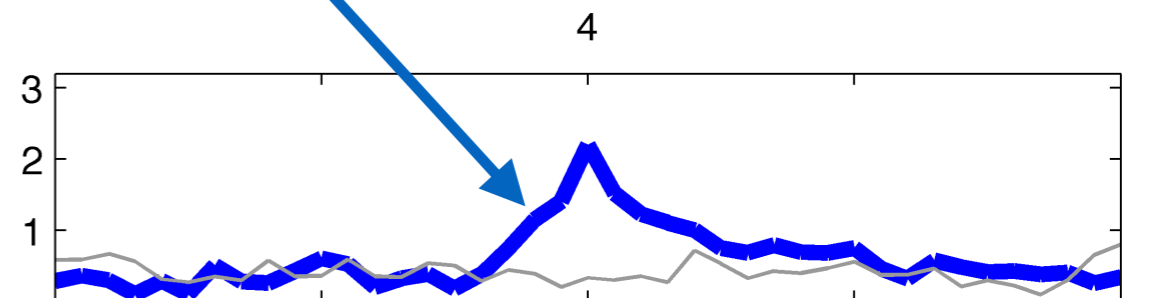
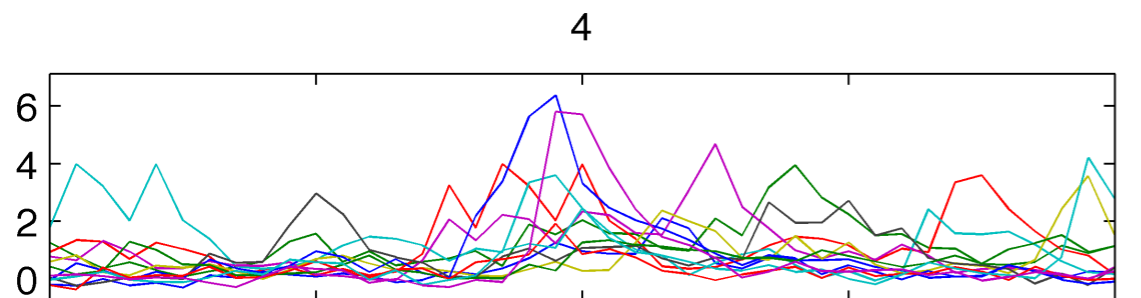
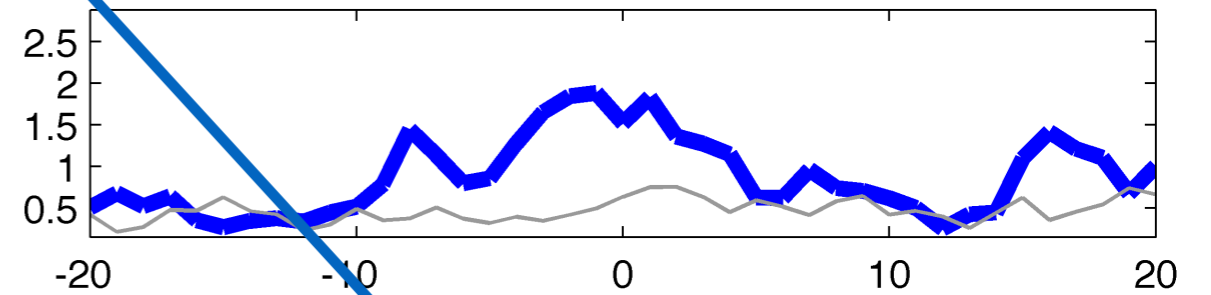
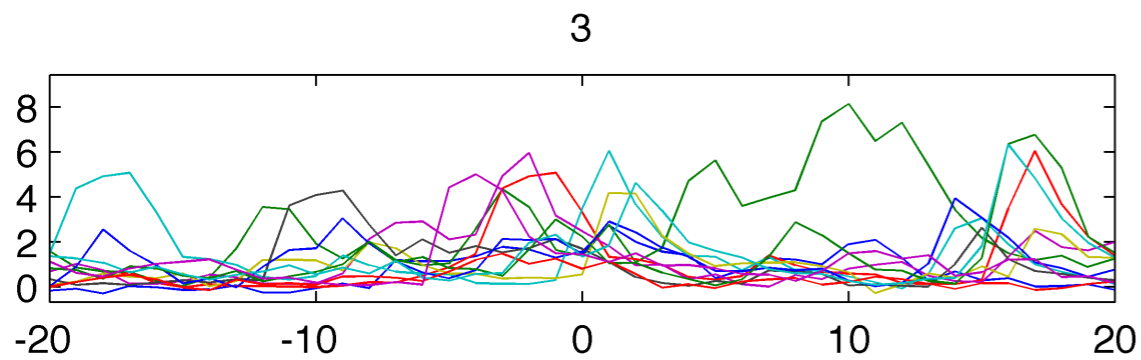
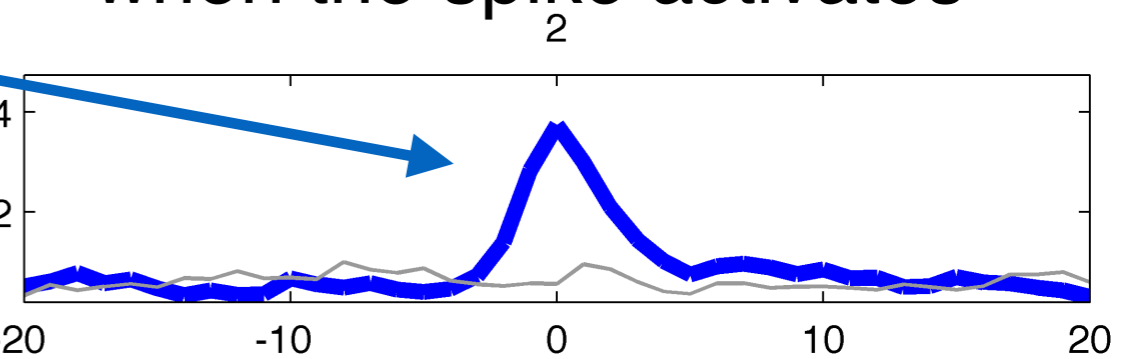
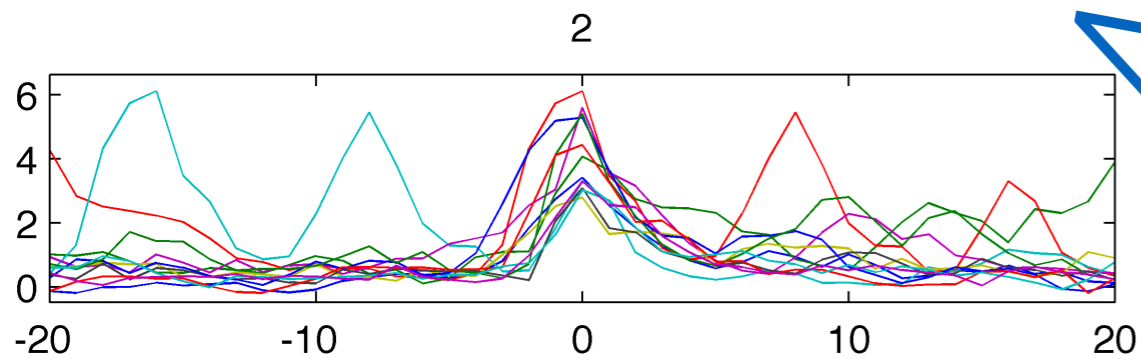


# Trigger-Based Averages

Threshold Amplitude: 2.5-6.0

**STA bump**

Non-random process activity  
when the spike activates



# Some Remarks

- All processes exhibit “spikes”
- Spikes are not all-or-none (amplitude varies)
- Activity seems mixed: spikes + “subthreshold” events
- Soma is less active than other processes – soma has fewer spikes, consistent with the need to receive enough inputs from processes to reach threshold
- Processes are more active than the soma – consistent with them being “closer” than soma to synaptic inputs



# Questions To Ask With The Model

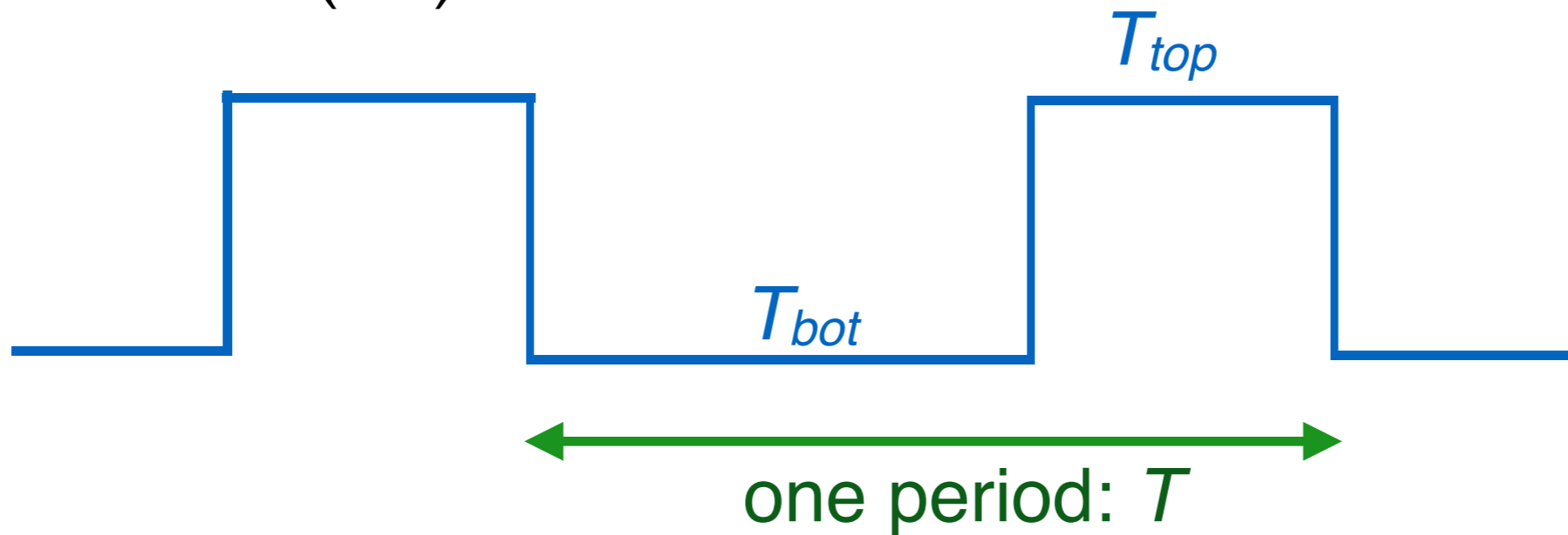
- Can we reproduce these results: processes are more active than soma; STAs of different ROIs are different, with bumps **preceding or following soma spikes**; activity is composed of events of various sizes; larger events are more likely to propagate further
- Do more active processes have STA bumps before soma spikes while less active processes have STA bumps following soma spikes?
- More generally (ambitiously) can we generate/explain the various shapes of the STAs observed experimentally

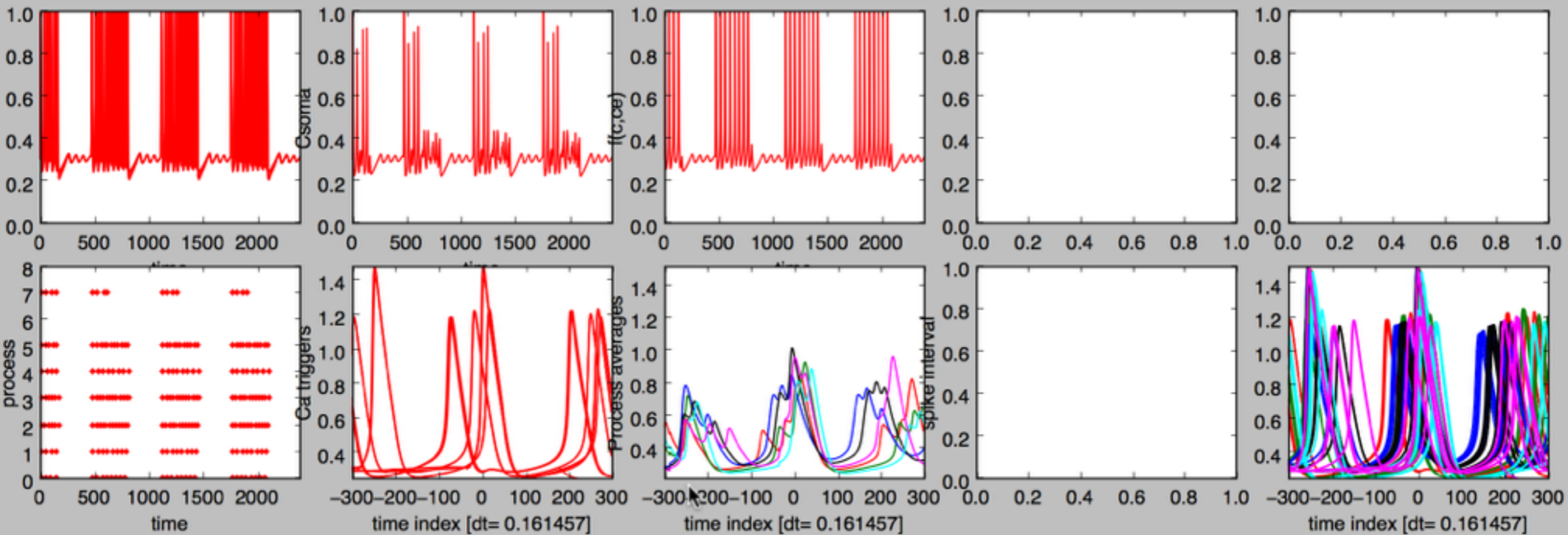
# Random Square Pulse

- To introduce randomness for “r” (neuronal activity), we turn “neurons” on and off to mimic experimental conditions
- We chose a square pulse
- We present a simple algorithm to generate square pulses [0 to 1 to 0] with control of frequency, and the time spent at 0 and 1.

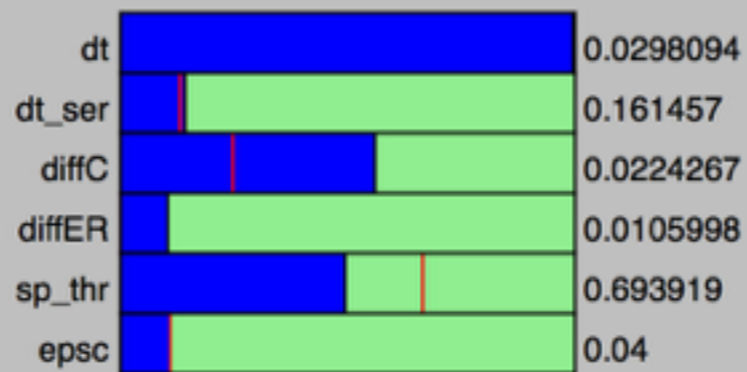
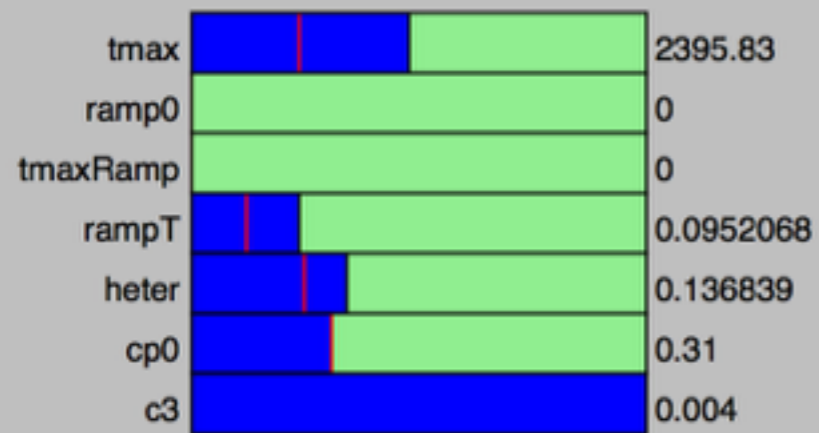
# Create a Square Pulse

- Control the period  $T$ , and the length of the top ( $T_{top}$ ) and bottom ( $T_{bot}$ )

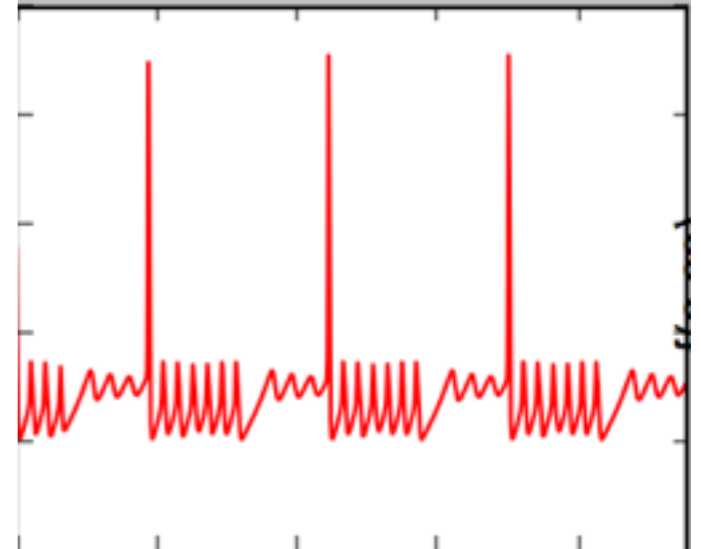
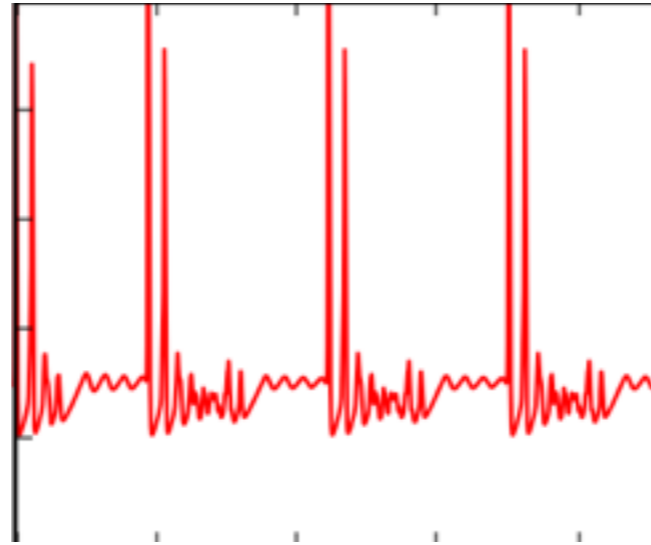
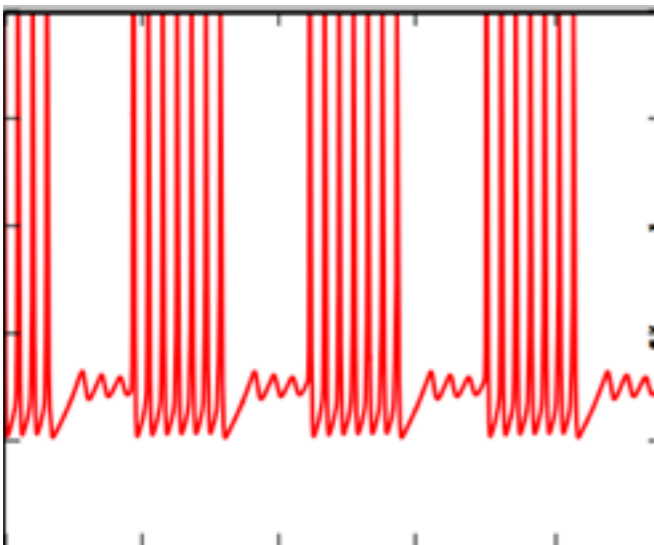
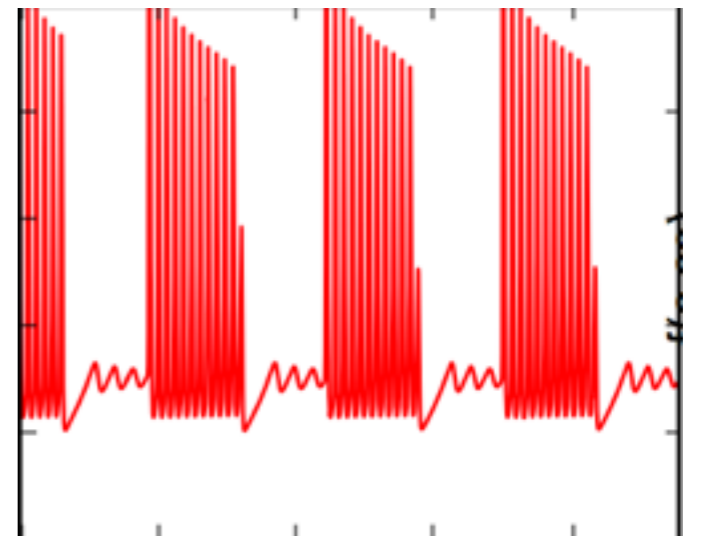
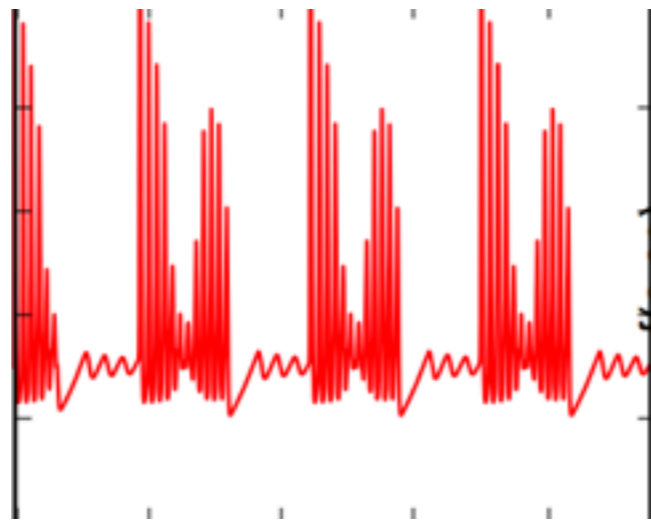
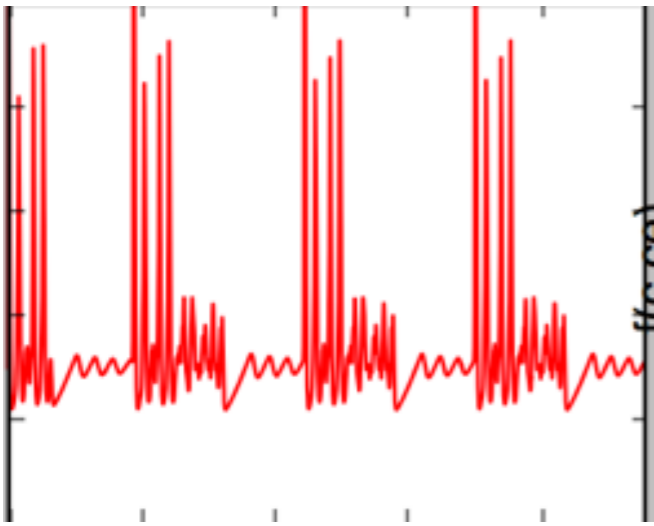
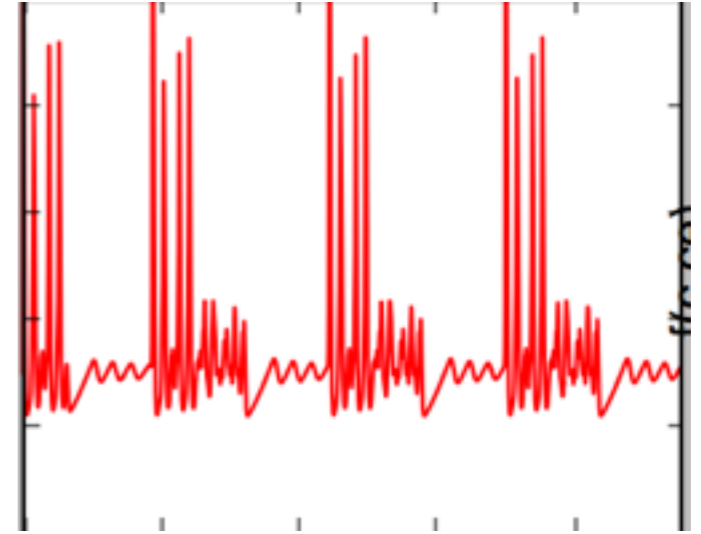
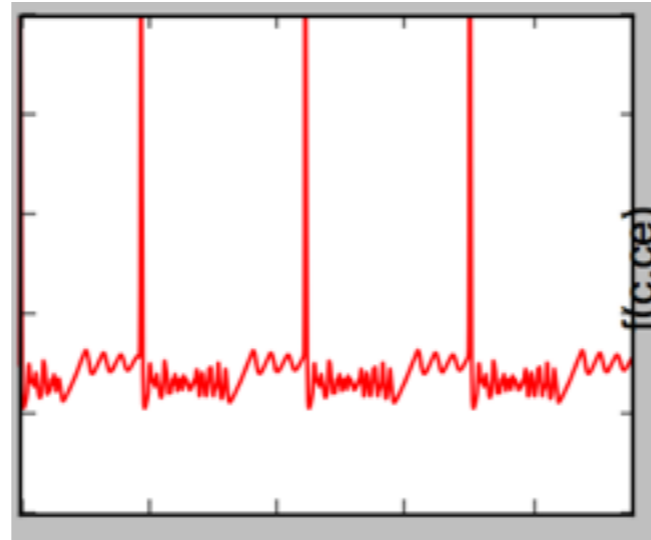
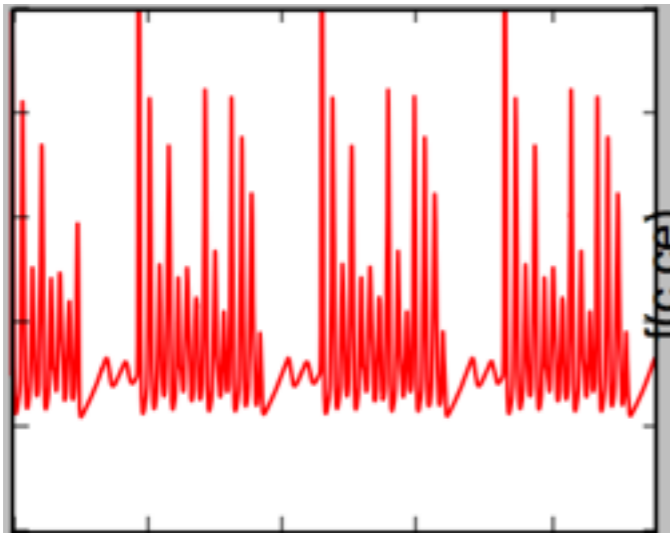




toggle



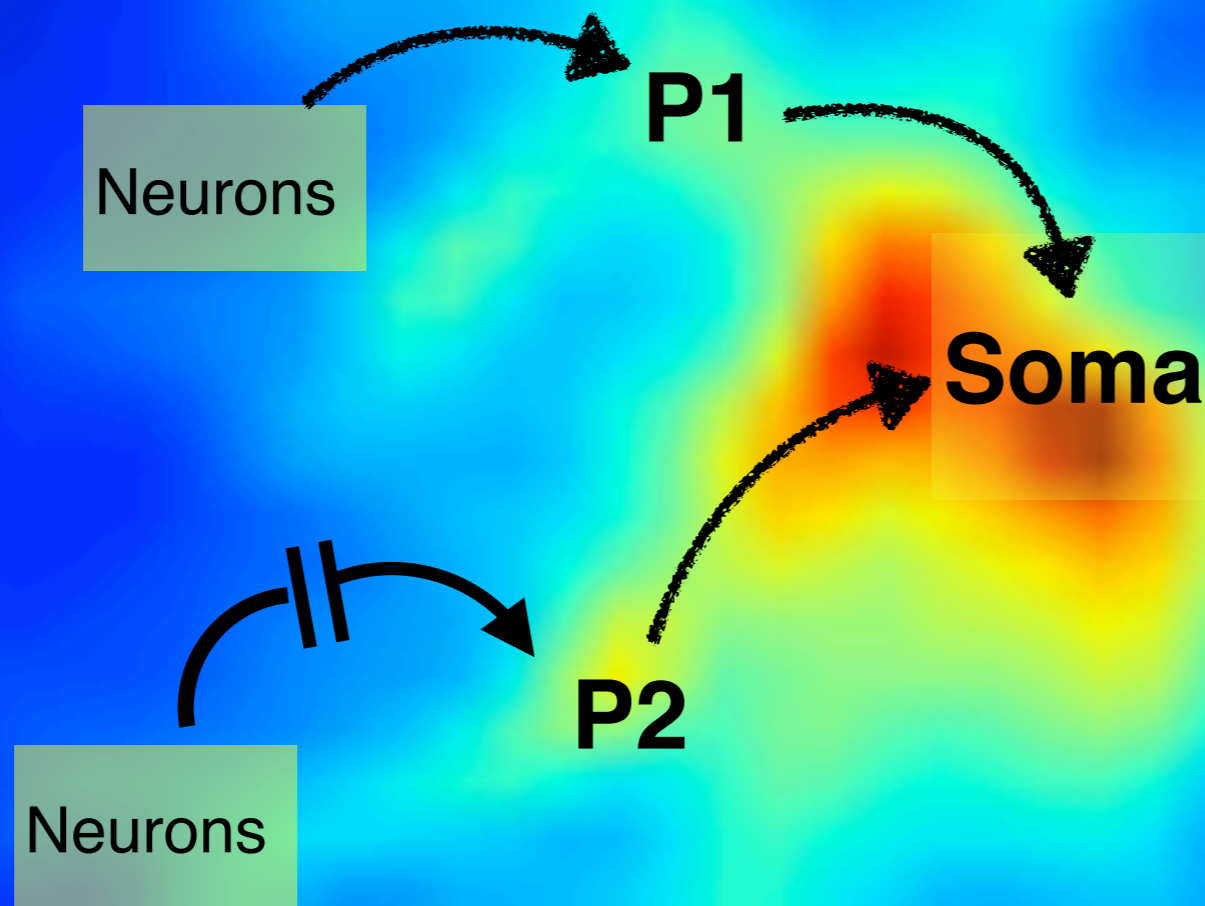
# Soma activity



# Effect of one process on another

- When a process spikes, what is its origin?
  - the soma?
  - neuronal input?
- We turn off process 0 to investigate

# Effect Of One Process On Another



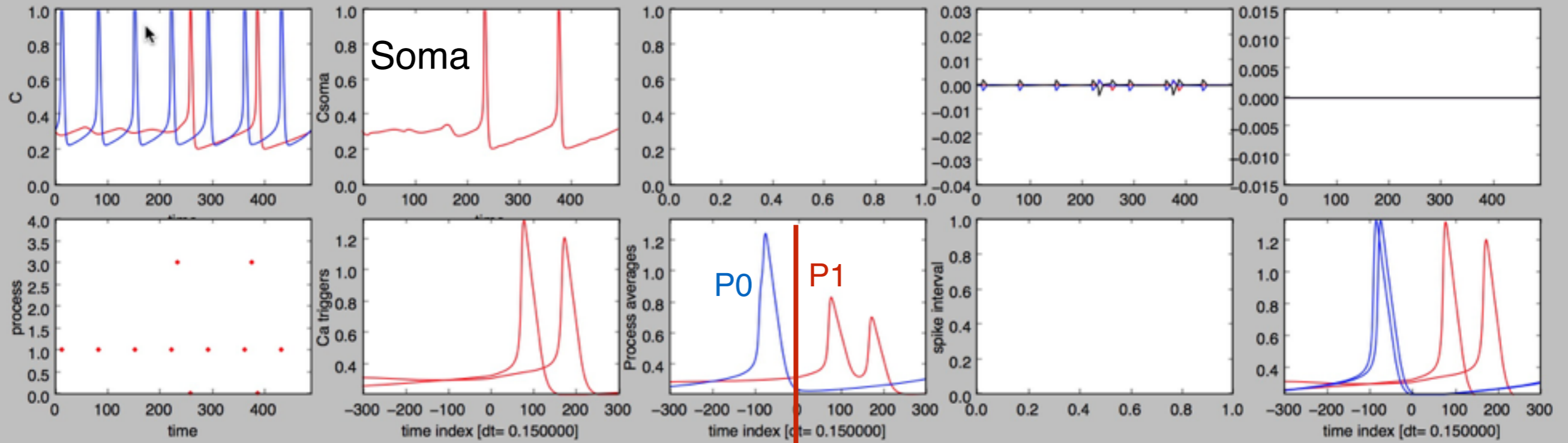
# Effect of one process on another

$$\frac{dCa_{p0}}{dt} = r_L + c_4 f() + \text{Diffusion}_0 - Ca_{p0}$$

$$\frac{dCa_{pi}}{dt} = r_L + r_{Amp} - c_4 f() + \text{Diffusion}_i - Ca_{pi}$$

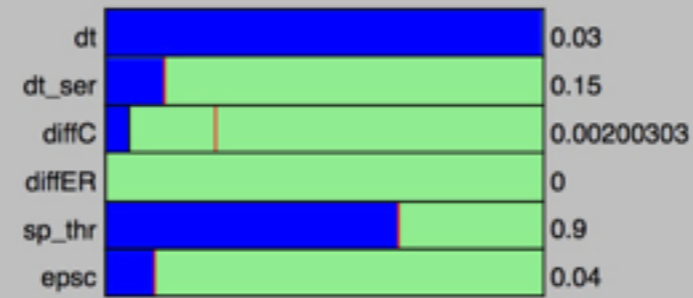
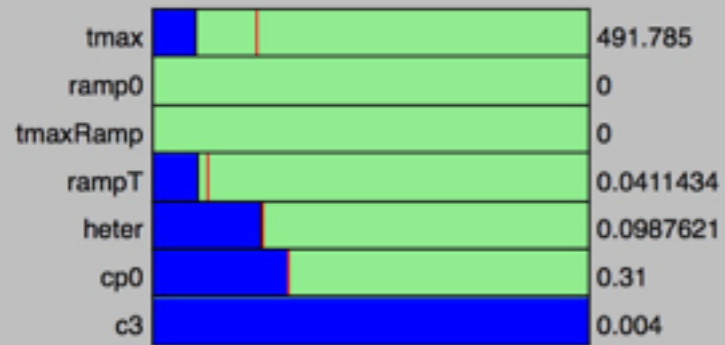
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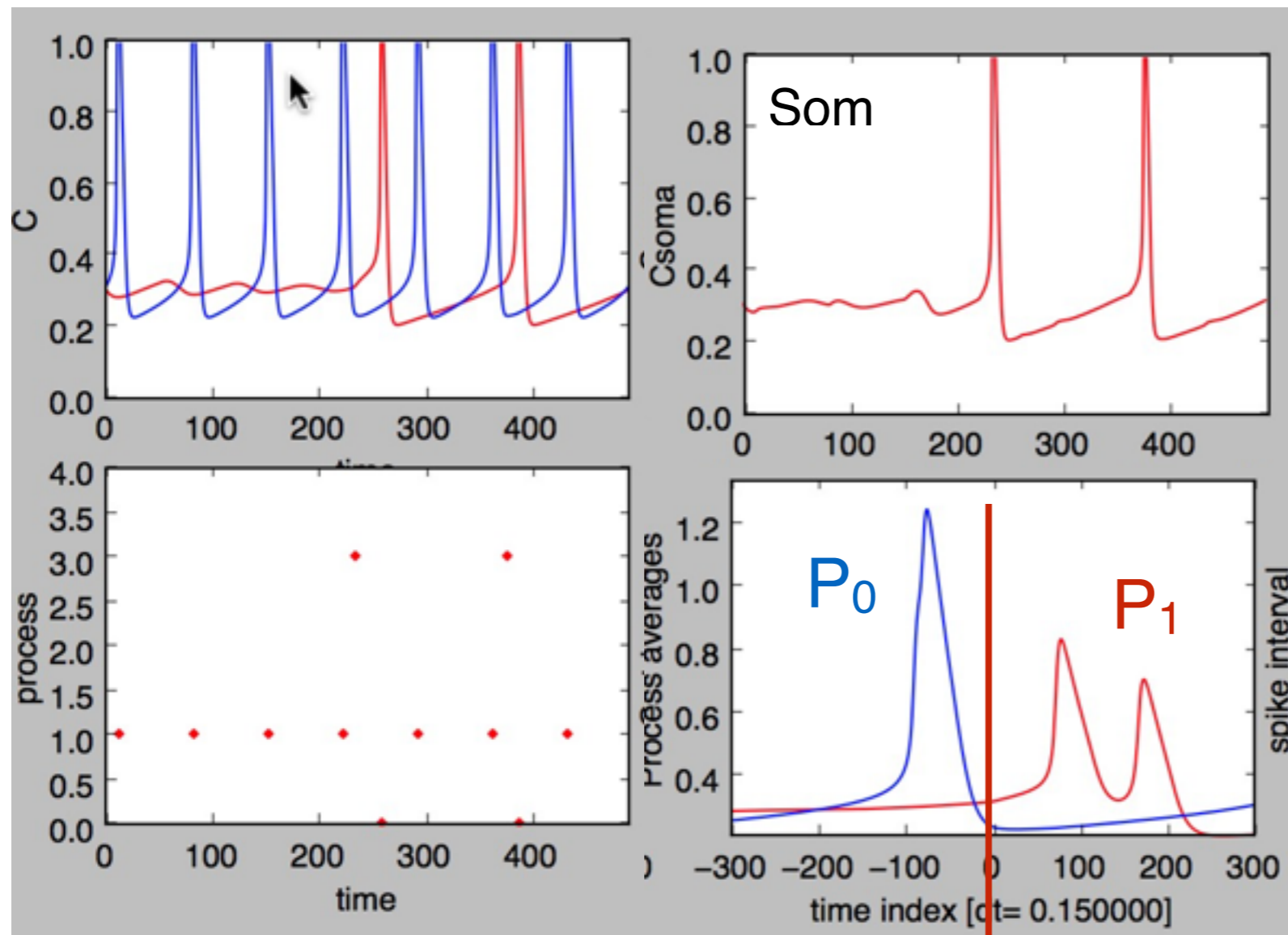




soma spike

toggle





**soma spike**

What happens in the real astrocyte?

What are the mechanisms responsible for spiking?

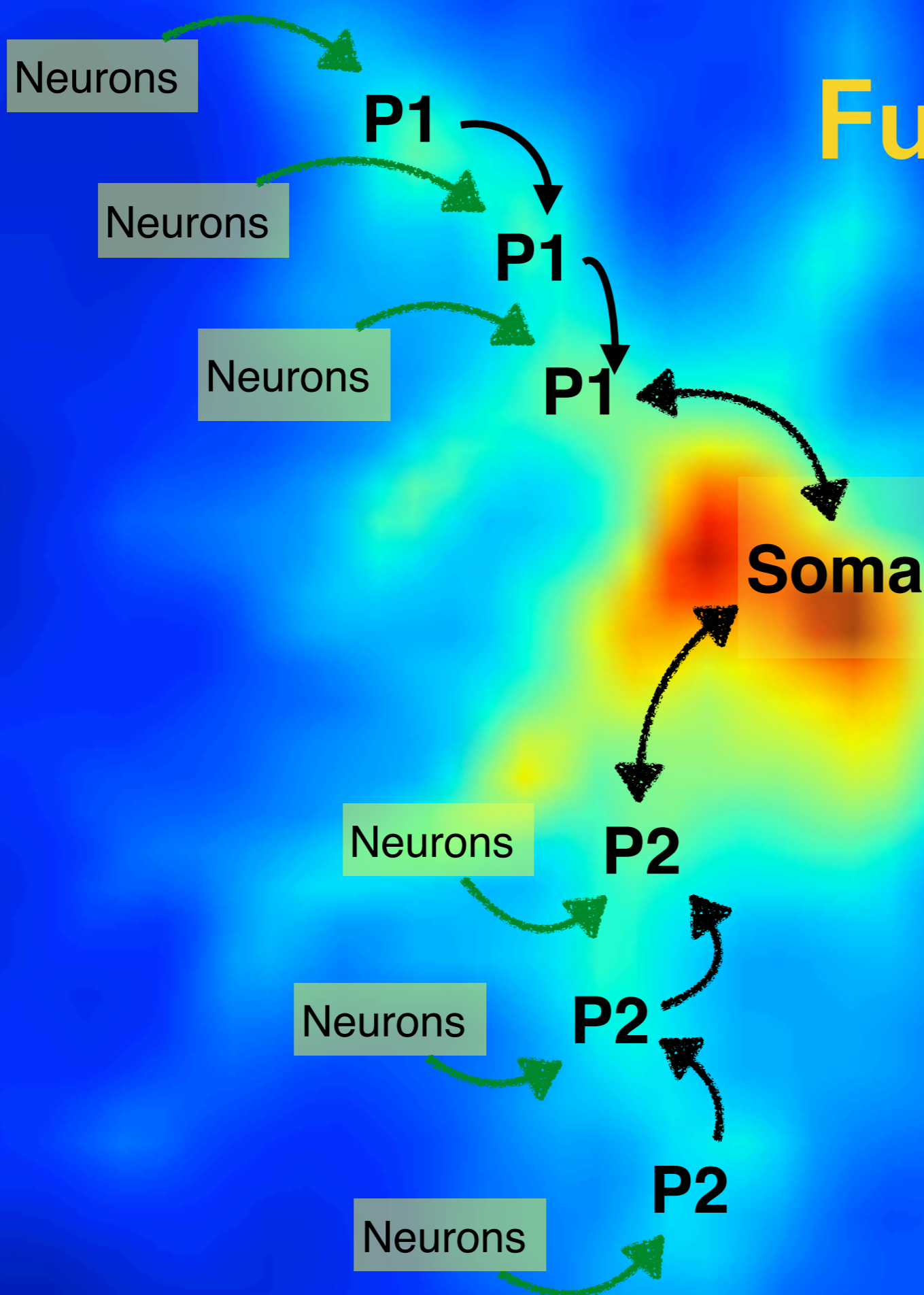
How are they modeled?

Can the model be predictive?

# Future Work

- Summarize our results
- Catalog and characterize spiking patterns as a function of randomness
  - at this stage, multiple runs with randomness produce rather different results
  - consider more complex models that take channel and IP3 production into account
  - take neuronal input into account

# Future work



Neurons are in the vicinity of the processes

# Fully Spatial Model

- Once we develop intuition with the models above, we can develop a fully spatial model
  - ODEs become PDEs with standard diffusion terms:

$$D\nabla^2 C_a$$

**Thank you!**

**Questions?**